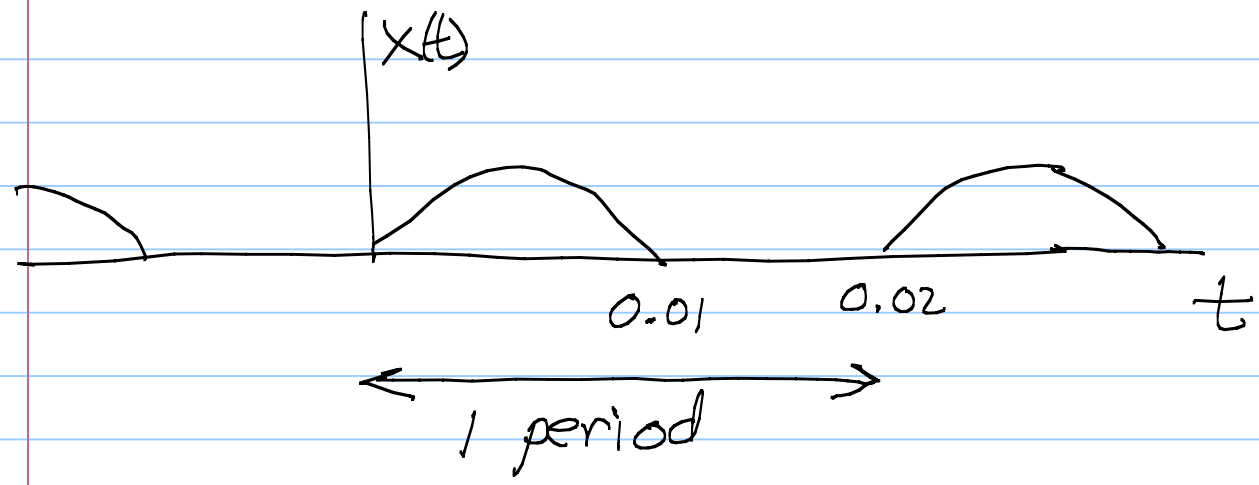
Fourier Series Example

Note Title 10/10/2004



$$x(t) = 220 \sin 2\pi 50 t \qquad 0 \le t \le 0.01$$
$$0 \qquad 0.01 \le t \le 0.02$$

$$a_k = \frac{1}{T_0} \int_0^{T_0} x(t)\, e^{-j\frac{2\pi k}{T_0} t}\, dt$$

$$= \frac{1}{0.02} \int_0^{0.01} 220 \sin 2\pi 50 t\, e^{-j\frac{2\pi k}{T_0} t}\, dt$$

We could look in integral tables to evaluate this, but let's do it a better way.

Use inverse Euler formula for the sin:

$$a_k = \frac{220}{0.02}\int_0^{0.01} \frac{1}{2j}\left(e^{j2\pi 50t} - e^{-j2\pi 50t}\right) e^{\frac{-j2\pi kt}{0.02}}\, dt$$

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$$\frac{1}{2j}\int_0^{0.01} e^{j2\pi 50(1-k)t}\, dt = \frac{1}{2j}\frac{1}{j2\pi 50(1-k)} e^{j2\pi 50(1-k)t}\Big|_0^{0.01}$$

$$= -\frac{1}{2\pi 50(1-k)}\left(e^{j\pi(1-k)} - 1\right)$$

2 similarly:

$$-\frac{1}{2j}\int e^{-j2\pi 50(1+k)t}\, dt = -\frac{1}{2\pi 50(1+k)}\left(e^{-j\pi(1+k)} - 1\right)$$

$$a_k = \frac{220}{0.02}\left(1 + 2\right)$$

$$= -(220)50\left(\frac{1}{2\pi 50(1-k)}e^{j\pi(1-k)} + \frac{1}{2\pi 50(1+k)}e^{-j\pi(1+k)}\right)$$

When k is odd: $e^{j\pi(1-k)} = e^{-j\pi(1+k)} = 1$

When k is even: $e^{j\pi(1-k)} = e^{-j\pi(1+k)} = -1$

So, for odd values of k:

$$a_k = \frac{-220}{2\pi}\left(\frac{1}{1-k} + \frac{1}{1+k}\right) = \frac{-220}{2\pi}\left(\frac{1+k+1-k}{(1+k)(1-k)}\right)$$

$$= \frac{220}{\pi}\cdot\frac{1}{k^2-1}$$

For even values:

$$a_k = \frac{-220}{2\pi}\left(-\frac{1}{1-k} - \frac{1}{1+k}\right) = -\frac{220}{\pi}\frac{1}{k^2-1}$$

We can combine these two results into one formula:

$$a_k = \frac{220}{\pi} \frac{1}{k^2 - 1} (-1)^{k+1}$$

Notice that the formula also applies for $k = 0$. The "DC" component is

$$a_0 = \frac{220}{\pi}$$