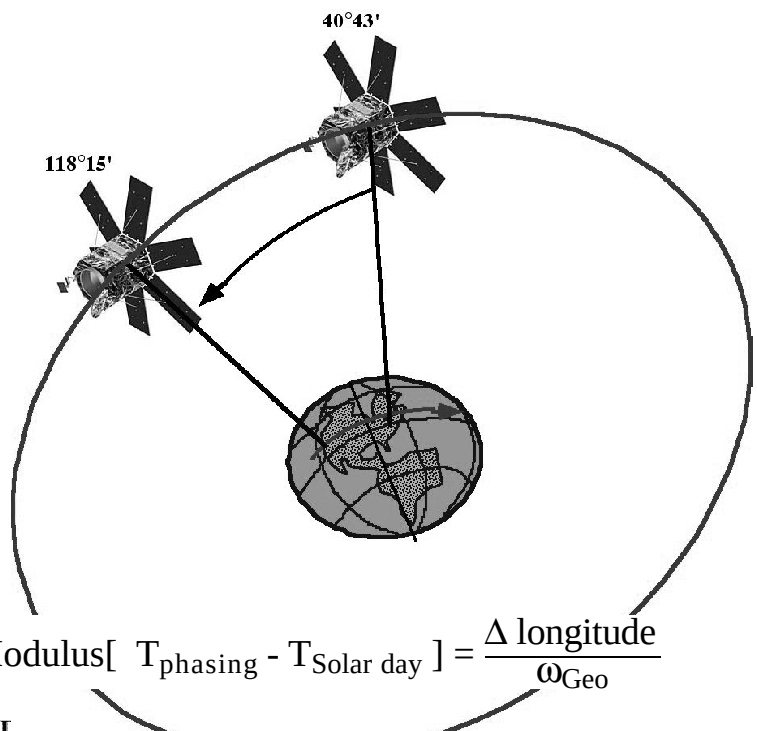
SS 3011
Space Technology and Applications

Applications of the Rocket-Equation
Calculating the Fuel Budget for an Orbital Phasing Maneuver of a GeoStationary Satellite

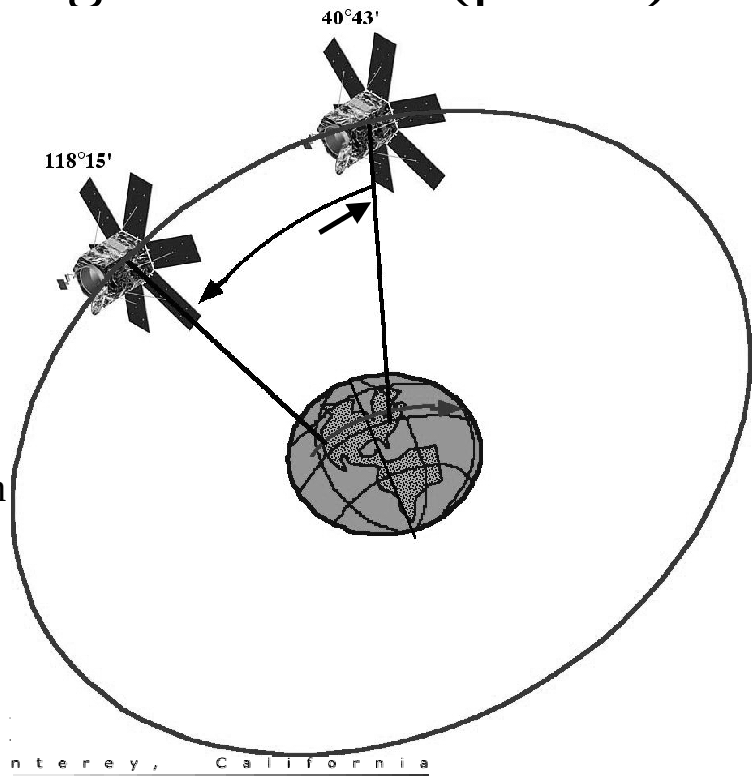
Phasing Maneuver

- Design a Orbital Maneuver that Allows Transfer of a GEO Synchronous Communication Satellite from the Longitude of New York City to the Longitude of Los Angeles



$$\text{Modulus}[\ T_{phasing} - T_{Solar\ day}\ ] = \frac{\Delta\ \text{longitude}}{\omega_{Geo}}$$

Phasing Maneuver (part 2)



• Design a Reverse Orbital Maneuver that Puts the Satellite Back to the Longitude of New York City after Mission Has been accomplished

What To Compute

• Compute

… Phasing Orbit Parameters
… Phasing Orbit Period
… Required ΔV_1, ΔV_2

• Assume R_{min} > 32,000 km
(to stay above Van Allen belts)

• Note: It may take Multiple orbits of Phasing Orbit to accomplish this task

What To Compute (cont'd)

- Compute
  - … Burn time for Transfer Orbit Insertion
  - … Burn Time for Final Orbit Insertion
  - … Required Fuel Budget for ΔV_1, ΔV_2

Parameters of the Problem

Solar Day: 23 hrs, 56 min, 4.1 seconds

Gravitational Parameter: $\mu = 3.986 \times 10^5 \frac{\text{km}^3}{\text{sec}^2}$

Longitude of New York City: 40 deg, 40 min West Longitude

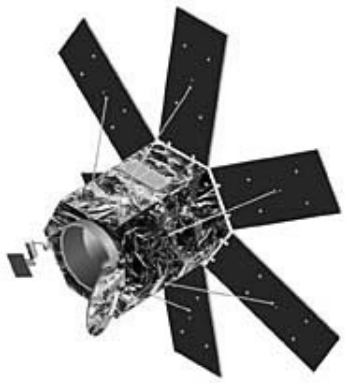
Longitude of Los Angeles: 118 deg, 15 min West Longitude

Parameters of the Problem (cont'd)

Specific Impulse

Fuel	Oxidizer	isp (s)
Liquid propellants		
Hydrogen (LH2)	Oxygen (LOX)	450
Kerosene (RP-4)	Oxygen (LOX)	260
Monomethyl hydrazine	Nitrogen Tetraoxide	310
Solid propellants		
Powered Al	Ammonium Perchlorate	270

Parameters of the Problem (Concluded)



- $F_{thruster}$ = 0.500 kNt
- Spacecraft mass

  1000 kg "Dry"

First Compute Radius of Geo Orbit

- **Kepler's Third law**

$$T = \frac{2\pi a^{3/2}}{\sqrt{\mu}}$$

$$T_{geo} = 23 \text{ hrs } 56 \text{ min } 4.1 \text{ sec} = 86164.1 \text{ sec}$$

$$a_{geo} = \left[\frac{\sqrt{\mu}\, T_{geo}}{2\pi}\right]^{2/3} =$$

$$\left[\frac{\sqrt{3.986 \times 10^5 \frac{km^3}{sec^2}} \times 86164.1 \text{ sec sec}}{2\pi}\right]^{2/3} = 42164.2 \text{ km}$$

Next Compute Angular Velocity of GEO Orbit

GEO orbit $\Rightarrow$ ω $\Rightarrow$ constant =

$$\frac{2\pi \text{ radians}}{86164.1 \text{ sec}} = .0000729212 \ \frac{\text{radians}}{\text{sec}}$$

Compute Linear Velocity of GEO Orbit

$$V_{GEO} = \sqrt{\frac{\mu}{a_{GEO}}} =$$

$$\sqrt{\frac{3.986 \times 10^5 \frac{km^3}{sec^2}}{42{,}164.2 \text{ km}}} = 3.0746 \frac{km}{sec}$$

Compute Required Time of Flight for Phasing Maneuver

Longitude of New York City: 40 deg, 40 min West Longitude

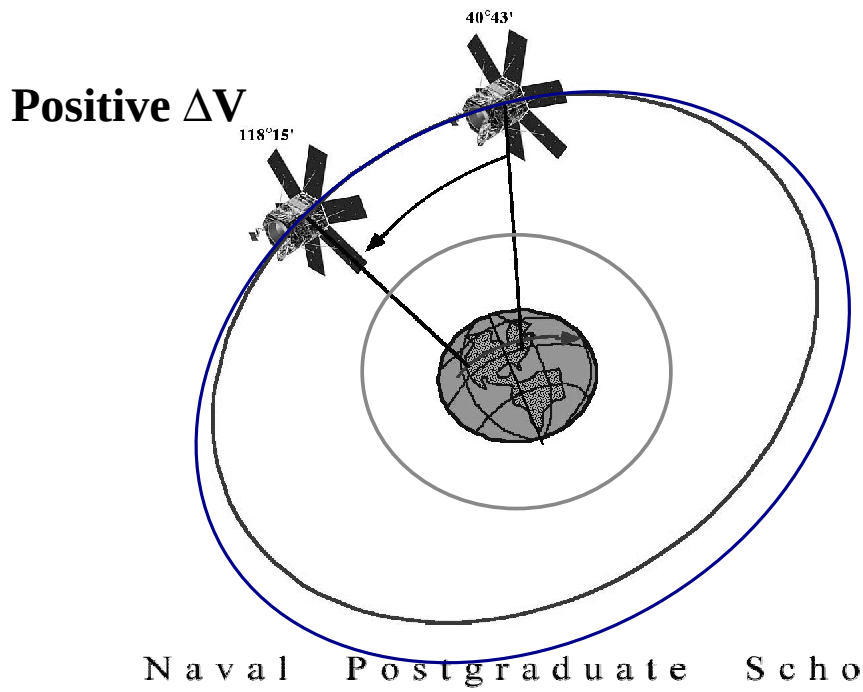
Longitude of Los Angeles: 118 deg, 15 min West Longitude

$$\Delta\nu = 118° \; 15' - 40° \; 40' = 77.5833 \; \hat{=} \; 1.3541 \text{ radians}$$

$$TOF = \frac{\Delta\nu}{\omega_{GEO}} = \frac{1.3541 \text{ radians}}{0000729212 \; \frac{\text{radians}}{\text{sec}}} =$$

$$18569.16 \text{ sec}$$

Use "Exterior" Phasing Orbit



Required Period for "Exterior" Phasing Orbit

$$\text{Modulus}[\ T_{phasing} - T_{Solar\ day}\ ] = \frac{\Delta\ \text{longitude}}{\omega_{Geo}}$$

$$T_{Phasing} = 86164.1 + 18569.2 = 104733.3 \text{ sec}$$

$$\frac{a_{phasing}}{a_{GEO}} = \left[\frac{T_{phasing}}{T_{GEO}}\right]^{2/3} = \left[\frac{104733.26 \text{ sec}}{86164.1}\right]^{2/3} = 1.13895$$

$$\Downarrow$$

$$a_{phasing} = 42{,}164.2 \text{ km} \times 1.13895 = 48{,}023.0 \text{ km}$$

Compute Eccentricity of the Phasing Orbit

$$a_{phasing}\left[1 - e_{phasing}\right] = R_{GEO} 42{,}164.2 \text{ km}$$

$$\Downarrow$$

$$e_{phasing} = 1 - \frac{R_{GEO}}{a_{phasing}} = 1 - \frac{42{,}164.2 \text{ km}}{48{,}023.0 \text{ km}} = 0.122$$

Do we need this?

Compute Perigee Velocity of Phasing Orbit

$$V_{perigee}^{(phasing)} = \sqrt{\frac{2\mu}{r_{perigee}} - \frac{\mu}{a_{phasing}}} =$$

$$\sqrt{2 \times \frac{3.986 \times 10^5 \frac{km^3}{sec^2}}{42{,}164.2 \text{ km}} - \frac{3.986 \times 10^5 \frac{km^3}{sec^2}}{48{,}023.0 \text{ km}}} = 3.2568 \frac{km}{sec}$$

Compute ΔV Required to Insert Spacecraft into Phasing Orbit

$$\Delta V_{phasing} = V_{perigee}^{(phasing)} - V_{GEO} =$$

$$3.2568 \frac{km}{sec} - 3.0746 \frac{km}{sec} = 0.1822 \frac{km}{sec}$$

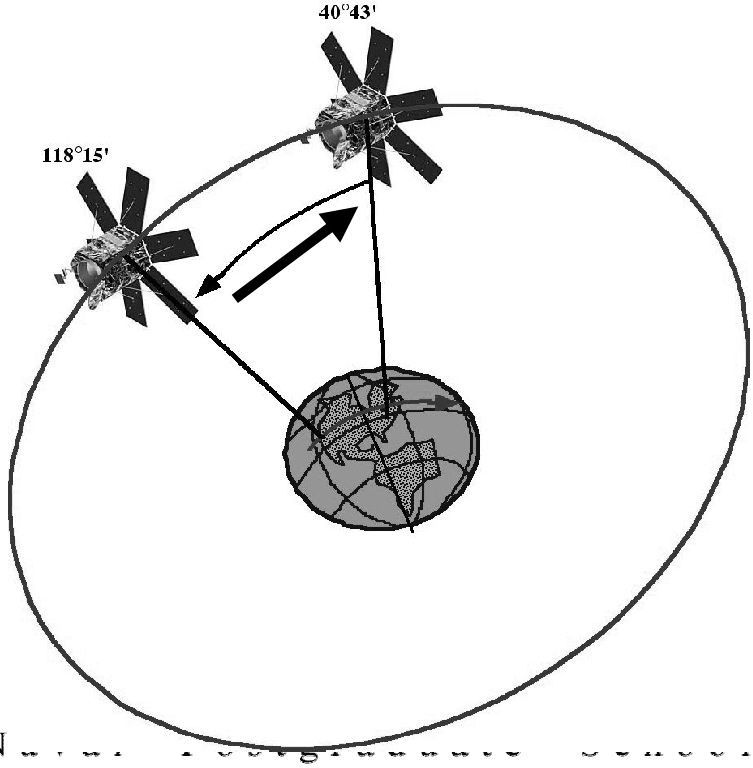
Compute ΔV Required to Insert Spacecraft into Final GEO Orbit

$$\Delta V_{Final} = V_{GEO} - V_{perigee}^{(phasing)}$$

$$3.0746 \frac{km}{sec} - 3.2568 \frac{km}{sec} = - 0.1822 \frac{km}{sec}$$

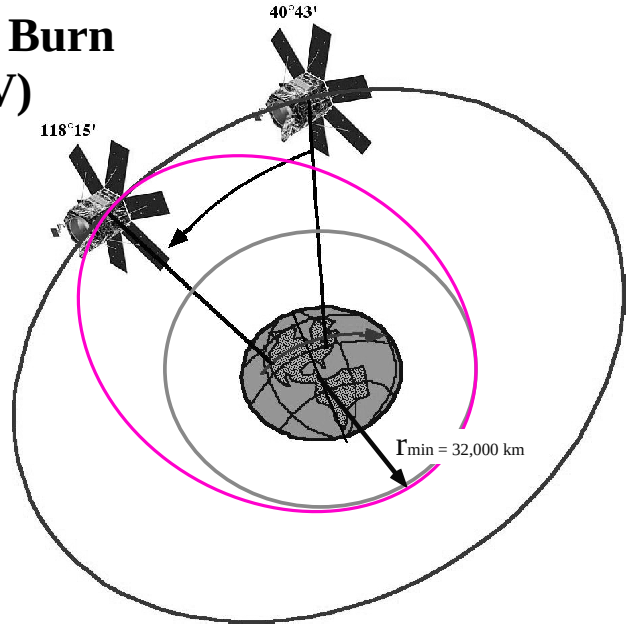
Retrograde Burn

Return Trip



Look "Interior" Phasing Orbit

Retro-grade Burn
(Negative ΔV)



Required Period for "Interior" Phasing Orbit

$$\text{Modulus}[\ T_{phasing} - T_{Solar\ day}\ ] = \frac{\Delta\ \text{longitude}}{\omega_{Geo}}$$

$$T_{Phasing} = 86164.1 - 18569.2 = 67594.9 \text{ sec}$$

Next Compute Semi-Major Axis of Phasing Orbit with Perigee at 32,000 km

(as per problem constraints)

$$a_{phasing} = \frac{r_{apogee} + r_{perigee}}{2} =$$

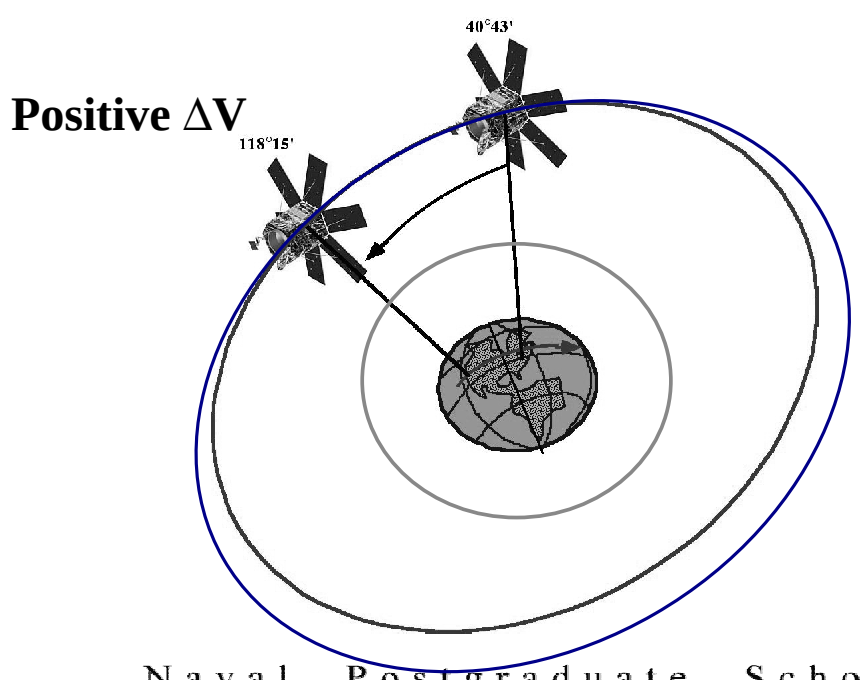
$$\frac{42164.2 \text{ km} + 32{,}000 \text{ km}}{2} = 37082.1 \text{ km}$$

Compute Period of Phasing Orbit with Perigee at 32,000 km (as per problem constraints)

$$T_{phasing} = \frac{2\pi a^{3/2}}{\sqrt{\mu}} = \frac{6.283185 \times [37082.1\ \text{km}]^{3/2}}{\sqrt{3.986 \times 10^5\ \frac{\text{km}^3}{\text{sec}^2}}} = 71065.3\ \text{sec}$$

- Allowable Minimum size interior orbit is far too big

Must use "Exterior" Phasing Orbit



Required Period for "Exterior" Phasing Orbit

$$\text{Modulus}[\ T_{phasing} - T_{Solar\ day}\ ] = \frac{\Delta\ \text{longitude}}{\omega_{Geo}}$$

$$T_{Phasing} = 2 \times 86{,}164.1 - 18{,}569.2 = 153{,}759 \text{ sec}$$

$$\frac{a_{phasing}}{a_{GEO}} = \left[\frac{T_{phasing}}{T_{GEO}}\right]^{2/3} = \left[\frac{153{,}759 \text{ sec}}{86164.1}\right]^{2/3} = 1.4712$$

$$\Downarrow$$

$$a_{phasing} = 42{,}164.2 \text{ km} \times 1.13895 = 62032.60 \text{ km}$$

Compute Perigee Velocity of Phasing Orbit

$$V_{perigee}^{(phasing)} = \sqrt{\frac{2\mu}{r_{perigee}} - \frac{\mu}{a_{phasing}}} =$$

$$\sqrt{2 \times \frac{3.986 \text{ x } 10^5 \ \frac{km^3}{sec^2}}{42{,}164.2 \text{ km}} - \frac{3.986 \text{ x } 10^5 \ \frac{km^3}{sec^2}}{62032.60 \text{ km}}} = 3.5329 \ \frac{km}{sec}$$

Compute ΔV Required to Insert Spacecraft into Phasing Orbit

$$\Delta V_{phasing} = V_{perigee}^{(phasing)} - V_{GEO} =$$

$$3.5329 \frac{km}{sec} - 3.0746 \frac{km}{sec} = .4583 \frac{km}{sec}$$

Compute ΔV Required to Insert Spacecraft into Final GEO Orbit

$$\Delta V_{Final} = V_{GEO} - V_{perigee}^{(phasing)}$$

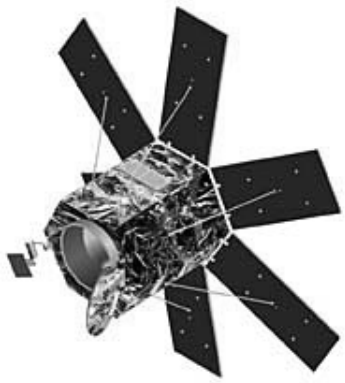
$$3.0746 \frac{km}{sec} - 3.5329 \frac{km}{sec} = - 0.4583 \frac{km}{sec}$$

Retrograde Burn

"Delta-Vee" Summary for Round Trip Maneuver

$$\begin{bmatrix} \Delta V_1 \\ \Delta V_2 \\ \Delta V_3 \\ \Delta V_4 \end{bmatrix} = \begin{bmatrix} -\ 0.1822 \ \frac{km}{sec} \\ -0.1822 \ \frac{km}{sec} \\ 0.4583 \ \frac{km}{sec} \\ -0.4583 \ \frac{km}{sec} \end{bmatrix}$$

Fuel Budgeting



- Spacecraft mass

  1000 kg "Dry"

- Start at Burn 4 and Work Backward

Weight of Vehicle at End of Maneuver Series

Burn 4 Fuel Consumption

$$\Delta V_{burn} = g_0 \, I_{sp} \, ln\left[\left(\frac{M_0}{M_{final}}\right)_{burn}\right]$$

$$\Downarrow$$

$$\boxed{M_{fuel} = \left[M_{dry} + M_{payload}\right]\left[e^{\left[\frac{\Delta V_{burn}}{g_0 I_{sp}}\right]} - 1\right]} =$$

$$1000\,\text{kg} \times \left[e^{\left[\frac{0.4583\,\frac{km}{sec}}{9.806\times 10^{-3}\,\frac{km}{sec^2}\times 310\,sec}\right]} - 1\right]$$

$$= 162.72 \text{ kg}$$

Burn 4 --> Burn Time

$$I_{sp} \equiv \frac{\left|\int_0^t \overline{F}\,dt\right|}{m_p} \approx \frac{\left|\overline{F}\right| T_{burn}}{m_p}$$

$$\Downarrow$$

$$T_{burn} = \frac{I_{sp} \times m_p}{\left|\overline{F}\right|} = \frac{310 \text{ sec} \times 162.72 \text{ kg}}{500\,\frac{\text{kg - m}}{\text{sec}^2}} = 100.9 \text{ sec}$$

Burn 3 Fuel Consumption and Burn Time

$$M_{fuel} = \left[M_{dry} + M_{payload}\right]\left[e^{\left[\frac{\Delta V_{burn}}{g_0 I_{sp}}\right]} - 1\right] =$$

Add in fuel Reserved for burn 4

$$1162.72\,\text{kg} \times \left[e^{\left[\frac{0.4583\,\frac{\text{km}}{\text{sec}}}{9.806\times10^{-3}\,\frac{km}{sec^2}\times 310\,sec}\right]} - 1\right]$$

$$= 189.20\ \text{kg}$$

$$T_{burn} = \frac{I_{sp}\times m_p}{|\overline{F}|} = \frac{310\ \text{sec}\times 189.20\ \text{kg}}{500\ \frac{\text{kg - m}}{\text{sec}^2}} = 117.3\ \text{sec}$$

Burn 2 Fuel Consumption and Burn Time

$$M_{fuel} = \left[M_{dry} + M_{payload}\right]\left[e^{\left[\frac{\Delta V_{burn}}{g_0 I_{sp}}\right]} - 1\right] =$$

Add in fuel Reserved for burns 3 and 4

$$1351.92\,\text{kg} \times \left[e^{\left[\frac{0.1822\,\frac{\text{km}}{\text{sec}}}{9.806\times10^{-3}\,\frac{km}{sec^2}\times 310\,sec}\right]} - 1\right]$$

$$= 83.51\ \text{kg}$$

$$T_{burn} = \frac{I_{sp}\times m_p}{|\overline{F}|} = \frac{310\ \text{sec}\times 83.51\ \text{kg kg}}{500\ \frac{\text{kg - m}}{\text{sec}^2}} = 51.78\ \text{sec}$$

Burn 1 Fuel Consumption and Burn Time

Add in fuel Reserved for burns 2, 3 and 4

$$M_{fuel} = \left[M_{dry} + M_{payload}\right]\left[e^{\left[\frac{\Delta V_{burn}}{g_0 I_{sp}}\right]} - 1\right] =$$

$$1435.43\,\text{kg} \times \left[e^{\left[\frac{0.1822\,\frac{\text{km}}{\text{sec}}}{9.806 \times 10^{-3}\,\frac{km}{sec^2} \times 310\,sec}\right]} - 1\right]$$

$$= 88.67\ \text{kg}$$

$$T_{burn} = \frac{I_{sp} \times m_p}{|\overline{F}|} = \frac{310\ \text{sec} \times 88.67\ \text{kg}}{500\ \frac{\text{kg - m}}{\text{sec}^2}} = 54.97\ \text{sec}$$

Propellant Burn Summary

$$\begin{bmatrix} \text{Burn}_1 & & 88.67\ \text{kg} \\ \text{Burn}_2 & = & 83.51\ \text{kg} \\ \text{Burn}_3 & & 189.20\ \text{kg} \\ \text{Burn}_4 & & 162.72\ \text{kg} \end{bmatrix}$$

- **524.1 kg total propellant consumed**

Burn-Time Summary

$$\begin{bmatrix} \text{Burn}_1 \\ \text{Burn}_2 \\ \text{Burn}_3 \\ \text{Burn}_4 \end{bmatrix} = \begin{bmatrix} 54.97 \text{ sec} \\ 51.78 \text{ sec} \\ 117.3 \text{ sec} \\ 100.9 \text{ sec} \end{bmatrix}$$

- 324..95 seconds **total burn life of engine expended**