

Signal Processing First

LECTURE #1 Sinusoids

READING ASSIGNMENTS

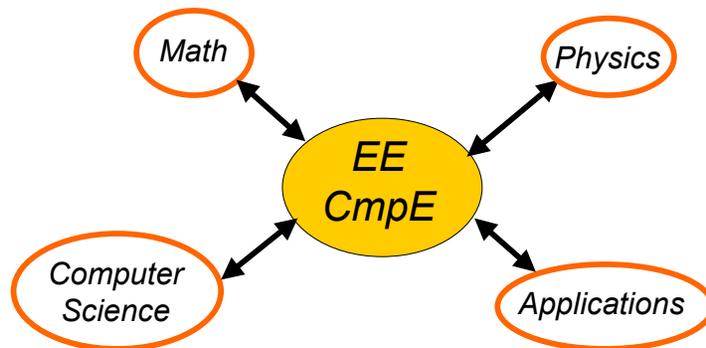
- This Lecture:
 - Chapter 2, pp. 9-17
- Appendix A: Complex Numbers
- Appendix B: MATLAB
- Chapter 1: Introduction

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CONVERGING FIELDS



COURSE OBJECTIVE

- Students will be able to:
- Understand **mathematical** descriptions of signal processing **algorithms** and express those algorithms as computer **implementations** (MATLAB)
- What are your objectives?

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WHY USE DSP ?

- Mathematical **abstractions** lead to generalization and discovery of new processing techniques
- Computer implementations are **flexible**
- Applications provide a **physical** context

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Fourier Everywhere

- Telecommunications
- **Sound & Music**
 - CDROM, Digital Video
- Fourier Optics
- X-ray Crystallography
 - Protein Structure & DNA
- Computerized Tomography
- Nuclear Magnetic Resonance: MRI
- Radioastronomy
- Ref: Prestini, "The Evolution of Applied Harmonic Analysis"

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LECTURE OBJECTIVES

- Write general formula for a "**sinusoidal**" waveform, or signal
- From the formula, plot the sinusoid versus time
- What's a **signal**?
 - It's a **function** of time, $x(t)$
 - in the mathematical sense

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TUNING FORK EXAMPLE

- CD-ROM demo 
- "A" is at 440 Hertz (Hz)
- Waveform is a SINUSOIDAL SIGNAL
- Computer plot looks like a sine wave
- This should be the mathematical formula:

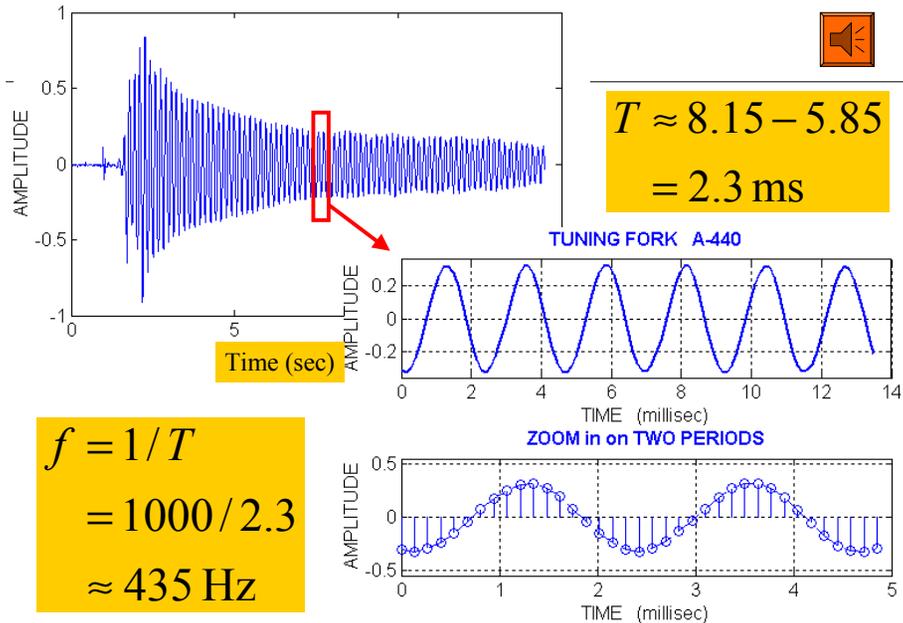
$$A \cos(2\pi(440)t + \varphi)$$

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TUNING FORK A-440 Waveform



SPEECH EXAMPLE

- More complicated signal (BAT.WAV) 
- Waveform $x(t)$ is NOT a Sinusoid
- Theory will tell us
 - $x(t)$ is approximately a sum of sinusoids
 - FOURIER ANALYSIS
 - Break $x(t)$ into its sinusoidal components
 - Called the FREQUENCY SPECTRUM

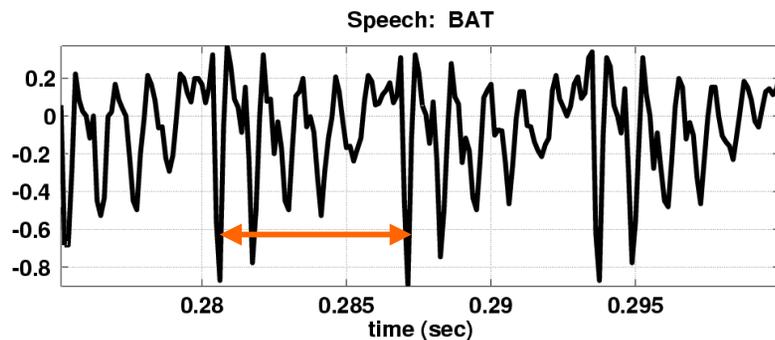
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Speech Signal: BAT

- Nearly Periodic in Vowel Region
 - Period is (Approximately) $T = 0.0065 \text{ sec}$



DIGITIZE the WAVEFORM

- $x[n]$ is a SAMPLED SINUSOID
 - A list of numbers stored in memory
- Sample at 11,025 samples per second
 - Called the SAMPLING RATE of the A/D
 - Time between samples is
 - $1/11025 = 90.7 \text{ microsec}$
- Output via D/A hardware (at F_{samp})

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STORING DIGITAL SOUND

- $x[n]$ is a SAMPLED SINUSOID
 - A list of numbers stored in memory
- CD rate is 44,100 samples per second
- 16-bit samples
- Stereo uses 2 channels
- Number of bytes for 1 minute is
 - $2 \times (16/8) \times 60 \times 44100 = 10.584$ Mbytes

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SINES and COSINES

- Always use the COSINE FORM

$$A \cos(2\pi(440)t + \varphi)$$

- Sine is a special case:

$$\sin(\omega t) = \cos(\omega t - \frac{\pi}{2})$$

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SINUSOIDAL SIGNAL

$$A \cos(\omega t + \varphi)$$

- **FREQUENCY** ω
 - Radians/sec
 - Hertz (cycles/sec)
- **AMPLITUDE** A
 - Magnitude
- **PERIOD** (in sec)
- **PHASE** φ

$$\omega = (2\pi)f$$

$$T = \frac{1}{f} = \frac{2\pi}{\omega}$$

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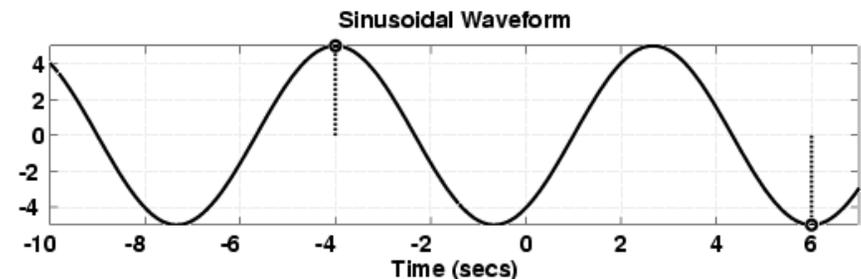
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EXAMPLE of SINUSOID

- Given the Formula

$$5 \cos(0.3\pi t + 1.2\pi)$$

- Make a plot



PLOT COSINE SIGNAL

$$5\cos(0.3\pi t + 1.2\pi)$$

- Formula defines A, ω , and ϕ

$$A = 5$$

$$\omega = 0.3\pi$$

$$\phi = 1.2\pi$$

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PLOTTING COSINE SIGNAL from the FORMULA

$$5\cos(0.3\pi t + 1.2\pi)$$

- Determine **period**:

$$T = 2\pi / \omega = 2\pi / 0.3\pi = 20/3$$

- Determine a **peak** location by solving

$$(\omega t + \phi) = 0 \Rightarrow (0.3\pi t + 1.2\pi) = 0$$

- Zero** crossing is T/4 before or after
- Positive & Negative peaks spaced by T/2

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PLOT the SINUSOID

$$5\cos(0.3\pi t + 1.2\pi)$$

- Use $T=20/3$ and the peak location at $t=-4$

