1. Given the probability distribution function

$$F_X(x) = \left(1 - e^{-x/5}\right)u(x)$$

Find:

a. $P[5 \le X \le 7]$

b. $P[X \ge 3]$

c. $P[X < 3]$

d. the probability density function, $f_X(x)$.

2. Let $Y = X^3$.

a. If the random variable X has a uniform pdf over the interval $[2, 5]$, find the mean and variance of Y.

b. If X is exponentially distributed with parameter $\alpha = 3$, find the mean and variance of Y.

3. The Maclaurin series expansion of the characteristic function in terms of moments of a random variable, X, is given by:

$$\Phi_X(j\omega) = \sum_{m=0}^{\infty} \frac{E\left[X^m\right]}{m!}(j\omega)^m$$

a. Find the charactersitc function of a Poisson random variable.

b. Obtain the moments of a Poisson random variable using the Maclaurin series expansion.

c. Using (b), find the mean and variance of a Poisson random variable.

4. a. Find the upper bound on the probability that a random variable is more than two standard deviations from its mean.

b. Find the exact probability that a uniformly distributed random variable is more than two standard deviations from its mean and compare with (a).

c. Find the exact probability that an exponentially distributed random variable is more than two standard deviations from its mean and compare with (a).

5. a. Find the probability generating function of a binomial random variable.

b. Use the probability generating function of (a) to find the mean and variance.

6. The random variable X has an exponential pdf given by $f_X(x) = 3e^{-3x}u(x)$.

a. Find the charactersitic function, $\Phi_X(\omega)$.

b. Use the characteristic function to find the meand and second moment of X

Find and sketch the pdf of random variable Y, where:

c. $Y = X^2$ d. $Y = |X|$ e. $Y = Xu(x)$ ($u(X)$ is a step function)