Staging Freight in a Crossdock

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Abstract. Most retail crossdocks stage freight outside of shipping doors to facilitate value-added processing and efficient loading. We describe a model for the queues of staged freight under two scenarios: staging pallets on the floor versus staging in a flow rack. Our results suggest that the value of flow rack is small in this context. We confirm the robustness of our analytical results with simulation and discuss implications for crossdock design.

Key words: Material handling, queueing, warehousing, crossdocking, simulation

1 Crossdocking

Crossdocking is a logistics technique that effectively eliminates the inventory-holding function of a warehouse while still allowing it to serve its consolidation and shipping functions. The idea is to transfer shipments directly from incoming to outgoing trailers without storage in between. Shipments typically spend less than 24 hours in a crossdock, sometimes less than an hour.

Crossdocks are essentially transshipment facilities to which trucks arrive with goods that must be sorted, consolidated with other products, and loaded onto outbound trucks. From a management perspective, crossdocking is a very complex enterprise, involving extensive coördination between the distributor and its suppliers and customers. The crossdock must know which products are arriving in which trucks at which times for which customers, and, if there is a high degree of consolidation, the crossdock must schedule trucks so as to avoid excessive congestion due to short term storage.

Crossdocks in the distribution industry exist in a wide variety of configurations. The simplest crossdock resembles a trucking terminal—a long, narrow building with doors around the perimeter. More complex facilities may have pallet rack for short term storage, conveyors for sorting and transporting packages, or automated storage devices (Napolitano, 2000).

Workers in a non-automated crossdock unload products, often on pallets, and transport them to outbound trailers. In the ideal case, the freight never touches the floor; however, workers often stage freight for a number of reasons:

- To allow value-added processing, such as pricing or labelling,
- To wait for other items of an order to arrive,
- To facilitate building tightly-packed loads, or
- To load in reverse order of delivery if there will be multiple stops.

To facilitate orderly staging, many crossdocks cordon off staging *lanes* in front of doors, into which workers place pallets for loading. We call the resulting queues *staging queues*. Because pallets do not automatically move forward when pallets at the head of the queue are removed, staging queues have unique properties, which we consider in this paper. Specifically, we compare the throughput of system that stages pallets on the floor with one that stages them in flow racks.

Several authors have addressed operational problems of crossdocking, including Peck (1983), Tsui and Chang (1990, 1992), Gue (1999), and Bartholdi and Gue (2000a), all of which address labor costs due to the placement of trailers into doors. Bartholdi and Gue (2000b) discuss the best shape for a crossdock facility; here, we discuss approaches to pallet queueing and the implications for crossdock design.

One way to classify crossdocks is according to *pre-* or *post*-distribution operations. In pre-distribution, shipments arrive at the crossdock with their destinations (retail outlets or other terminals) already determined and labeled. Workers simply take shipments directly to outbound trailers. In post-distribution, workers at the crossdock assign destinations to products. For example, if a load of 40 pallets of household fans arrives, workers might assign two pallets to Store A, 1 pallet to Store B, and so on.

Pre-distribution crossdocking is more difficult for retail firms because it requires excellent information sharing and communications with vendors. Vendors typically afix store labels and even price tags to the products. With respect to material handling, pre-distribution crossdocking is easier because there is no need to put product on the floor—it can go right into the trailer. In post-distribution crossdocks, workers must stage freight in order to assign destination doors, and this leads to double handling and a need for more floor space.

Whether an operation is pre-distribution or post-distribution has important implications for the design of the facility. Because pre-distribution means that shipments are not staged for as long, there is less need for floor space, and the facility can be narrow. Narrower docks are more efficient because workers need not travel as far to transfer freight (Bartholdi and Gue, 2000b). In post-distribution, the dock must be wide enough to allow workers to stage freight between receiving and shipping.

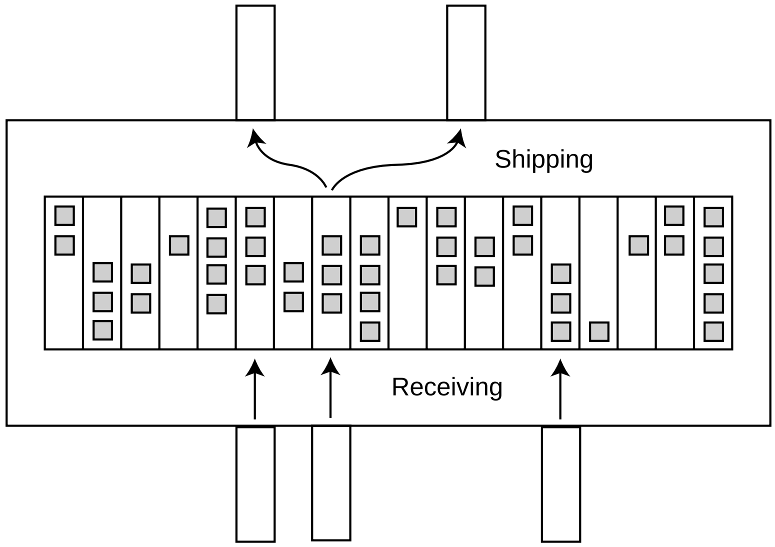

Figure 1: A single-stage crossdock. Workers put pallets in lanes corresponding to the receiving doors.

We can also classify crossdocks according to the type of staging: single-stage, two-stage, or free staging. In a single-stage crossdock (see Figure 1), workers unload pallets and put them into queues corresponding to receiving doors or shipping doors. For post-distribution crossdocking, queues often correspond to receiving doors because the destination of each pallet is unknown upon arrival; for pre-distribution, queues can be according to receiving or shipping doors.

Figure 2 illustrates a two-stage crossdock, similar to that used by Costco in Tracy, CA. A two-stage system has the advantage of allowing workers in shipping to pick from among several pallets in a shipping queue (which results in more tightly packed loads), while still allowing value-added processing by workers in receiving. The disadvantage, of course, is that pallets are handled an additional time, and the crossdock must be wider to accommodate the additional queue, resulting in additional labor cost due to travel.

Crossdocks in the less-than-truckload (LTL) trucking industry have a free staging area outside each shipping door; that is, an area not amenable to placing pallets at one end and pulling from the other. This method is necessary because LTL crossdocks have shipping doors on both sides of the dock, and the docks are typically very narrow, which allows access to the staging area from only one side. LTL operations do not lend themselves to staging queues as we define them due to the great variety of freight they must accommodate.

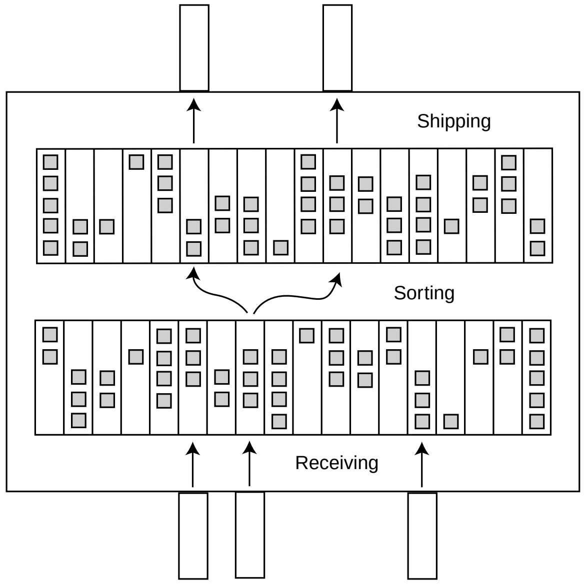

Figure 2: Representation of a two-stage crossdock operated by Costco in Tracy, CA. Workers put pallets in lanes corresponding to the receiving doors; a second team of workers sorts pallets into shipping lanes, from which a final team loads them onto outbound trailers.

2 A throughput model

Conceptually, a single-stage crossdock may be viewed as a network of queues: shipments are customers that arrival in bulk (on a trailer); workers serve (sort) them, placing them into staging queues, from which another set of workers serves them (delivers into outbound trailers), and they depart the system in bulk. Our goal is to determine how the staging queue affects throughput, and consequently how staging queues should influence design.

2.1 Staging queues

When workers place a pallet in a staging lane they place it as far forward as possible. Working from the other side of the queue, workers pull pallets out of the lane and deliver them to destination trailers. Because the lanes are narrow, workers on the shipping side usually pull the forwardmost pallet from the queue, and workers on the receiving side place pallets in the forwardmost open position from the rear. We call this a *staging queue* (Figure 3).

We model the staging queue as a continuous time Markov chain, beginning with the following lemma.

Lemma 1 *Pallets in a staging queue must be contiguous.*

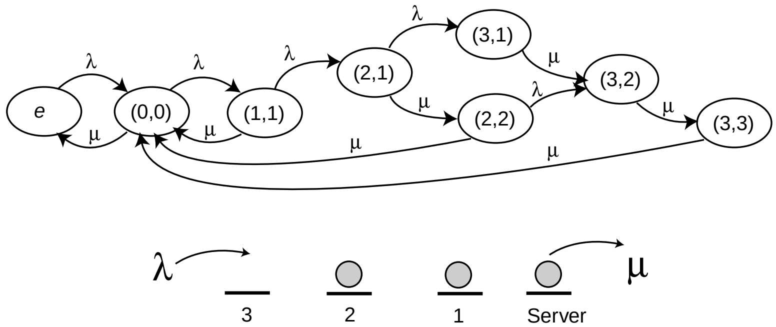

Figure 3: A 3-pallet staging queue. The bottom figure illustrates material movement: pallets arrive at rate λ and are placed in the forwardmost open position. Workers pull pallets from the queue at rate μ. The system is currently in state $(2,1)$. The top figure illustrates transitions between states of the continuous time Markov chain.

This is because pallets enter the queue from the rear and occupy the forwardmost position, and only the forwardmost pallet from the front may be served. Formally, we say the system is in state (i,j) when the rearwardmost occupied position is i and the forwardmost occupied position is j (therefore $i \geq j$.) If no positions are occupied and the server is busy, the system is in state $(0,0)$; otherwise, the server is idle and the system is empty and in state e.

Let π_{ij} be the steady state probability that the system is in state (i,j). Figure 3 illustrates the transition probabilities for a queue size of 3. We see that,

$$\begin{aligned}
\lambda\pi_e &= \mu\pi_{00}, \\
(\lambda+\mu)\pi_{00} &= \lambda\pi_e + \mu(\pi_{11} + \pi_{22} + \pi_{33}), \\
(\lambda+\mu)\pi_{11} &= \lambda\pi_{00}, \\
(\lambda+\mu)\pi_{21} &= \lambda\pi_{11}, \\
(\lambda+\mu)\pi_{22} &= \mu\pi_{21}, \\
\mu\pi_{31} &= \lambda\pi_{21}, \\
\mu\pi_{32} &= \lambda\pi_{22} + \mu\pi_{31}, \\
\mu\pi_{33} &= \mu\pi_{32}, \text{ and} \\
\pi_e + \sum_{i,j}\pi_{ij} &= 1.
\end{aligned}$$

The transition probabilities are

$$\pi_e = \frac{\mu}{\lambda}\pi_{00}, \tag{1}$$

$$\pi_{11} = \frac{\lambda}{\lambda+\mu}\pi_{00}, \tag{2}$$

$$\pi_{21} = \frac{\lambda^2}{(\lambda+\mu)^2}\pi_{00}, \tag{3}$$
$$\pi_{22} = \frac{\lambda^2\mu}{(\lambda+\mu)^3}\pi_{00}, \tag{4}$$
$$\pi_{31} = \frac{\lambda^3}{\mu(\lambda+\mu)^2}\pi_{00}, \tag{5}$$
$$\pi_{32} = \frac{2\lambda^3\mu+\lambda^4}{(\lambda+\mu)^3}\pi_{00}, \tag{6}$$
$$\pi_{33} = \frac{2\lambda^3\mu+\lambda^4}{(\lambda+\mu)^3}\pi_{00}, \tag{7}$$

where

$$\pi_{00} = \left(1 + \frac{\mu}{\lambda} + \frac{\lambda}{\lambda+\mu} + \frac{\lambda^2}{(\lambda+\mu)^2} + \frac{\lambda^2\mu}{(\lambda+\mu)^3} + \frac{\lambda^3}{\mu(\lambda+\mu)^2} + \frac{4\lambda^3\mu+2\lambda^4}{\mu(\lambda+\mu)^3}\right)^{-1}.$$

Arrivals are served anytime the last position in the queue is occupied, so the effective system throughput is $\lambda_{\text{eff}} = (1 - \pi_{31} - \pi_{32} - \pi_{33})\lambda$.

If we add a position to the example staging queue, equations 1–4 are the same; we need only derive new equations for what were previously blocking states (π_{3j}), the new blocking states (π_{4j}), and recompute π_{00} to obtain the probabilities. Following is the general result.

Theorem 1 *In a staging queue with n positions, steady state probabilities π_{ij} are, for non-blocking states, $\pi_e = (\mu/\lambda)\pi_{00}$ and $\pi_{ij} = r_{ij}\, a_{ij}\, \pi_{00}$, where*

$$\begin{aligned} r_{ij} &= r_{i-1,j} + r_{i,j-1}, \ \textit{(where } r_{ij} = 0 \textit{ for } i < j,\ r_{i0} = 0, \textit{ and } r_{i1} = 1\textit{)}, \\ a_{ij} &= \left(\frac{\lambda}{\lambda+\mu}\right)^i \left(\frac{\mu}{\lambda+\mu}\right)^{j-1} \ \textit{(for } i = 1 \ldots n-1,\ j = 1 \ldots i\textit{)}, \\ \pi_{00} &= \frac{1}{1 + \mu/\lambda + \sum_{ij} a_{ij}}; \end{aligned}$$

and for blocking states,

$$\begin{aligned} \pi_{n1} &= (\lambda/\mu)\pi_{n-1,1}, \\ \pi_{ni} &= \lambda\pi_{n-1,i} + \mu\pi_{n,i-1}, \ \textit{(for } i = 2 \ldots n-1\textit{), and} \\ \pi_{nn} &= \pi_{n,n-1}. \end{aligned}$$

Proof. For transition probabilities π_e and π_{00} we appeal to the state diagram directly. For the remaining non-blocking states, we prove the result by induction. Consider the state $(1,1)$ in the state diagram,

$$\begin{aligned}\pi_{11} &= \frac{\lambda}{\lambda+\mu}\pi_{00}\\ &= (r_{10}+r_{01})\frac{\lambda}{\lambda+\mu}\pi_{00},\end{aligned}$$

which is the result.

Now assume the result is true for $\pi_{i,j-1}$ and $\pi_{i-1,j}$. Notice from the state diagram that for all non-blocking states except state e the relationship $(\lambda+\mu)\pi_{ij} = \mu\pi_{i,j-1} + \lambda\pi_{i-1,j}$ holds, where $\{\pi_{ij} = 0 : i < j \text{ or } j = 0\}$. Then,

$$\begin{aligned}(\lambda+\mu)\pi_{ij} &= \mu\pi_{i,j-1} + \lambda\pi_{i-1,j}\\ \pi_{ij} &= \frac{\mu}{\lambda+\mu}\pi_{i,j-1} + \frac{\lambda}{\lambda+\mu}\pi_{i-1,j}\\ &= \frac{\mu}{\lambda+\mu}\left(\frac{r_{i,j-1}\lambda^i\mu^{j-2}}{(\lambda+\mu)^{i+j-2}}\right)\pi_{00} + \frac{\lambda}{\lambda+\mu}\left(\frac{r_{i-1,j}\lambda^{i-1}\mu^{j-1}}{(\lambda+\mu)^{i+j-2}}\right)\pi_{00}\\ &= (r_{i-1,j}+r_{i,j-1})\frac{\lambda^i}{(\lambda+\mu)^i}\frac{\mu^{j-1}}{(\lambda+\mu)^{j-1}}\pi_{00}.\end{aligned}$$

For blocking states, we appeal directly to the state diagram.

□

Theorem 1 leads to the following corollary:

Corollary 1 *The effective throughput for an n-position staging queue is* $\lambda_{eff} = (1 - \sum_j \pi_{nj})\lambda$.

The model has several limitations with respect to crossdocking operations in practice. For example, arrivals to a blocked queue take action to clear the block, such as notifying workers in the shipping area or clearing the block themselves. Also, some crossdocks have two staging lanes per trailers, giving workers in receiving two queues into which they can drop a pallet. We also ignore the effects of travel distance: arrival and service rates change to some degree as pallets in the queue move. Despite these drawbacks, we believe the model gives insight into how staging queues behave, and it allows us to compare them with queues formed by flow racks.

2.2 Flow rack

How much worse is the staging queue than flow rack? Pallets in a flow rack automatically roll to the front of the queue. We can describe the state of this queue with a single variable because the queue always fills from the front. Let a queue with i pallets in it be in state i. It is easy to show that for flow rack with n pallet positions and $\rho = \lambda/\mu$, $\pi_i = \rho^i\pi_0$ and $\pi_0 = (1+\rho+\rho^2+\ldots+\rho^n)^{-1}$. As before, the system is producing whenever the queue is not empty, so effective system throughput is $\lambda_{\text{eff}} = (1-\pi_n)\lambda$.

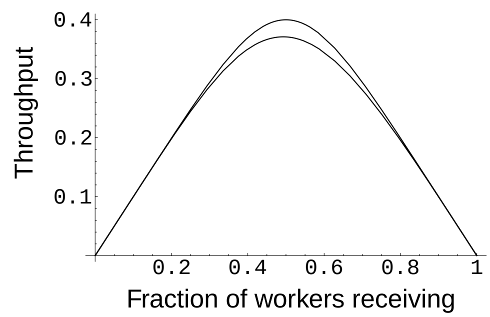

Figure 4: Theoretical throughput for 3-position staging and flow rack queues as the allocation of workers varies. The top curve is for the flow rack; the bottom for the staging queue.

2.3 Optimal allocation of work

The throughput of the crossdock depends on the buffer size, the queue policy, and the number and allocation of workers.

From the steady state probabilities, we know that π_{nj} depends on λ and μ, and these rates depend on the allocation of workers to either receiving or shipping. Suppose we have n workers to allocate between receiving and shipping. Figure 4 shows the throughput of both the flow rack and staging queues as the fraction of workers in receiving (i.e., $\lambda/(\lambda+\mu)$) varies between 0 and 1. The optimal allocation for the staging queue has slightly more workers in shipping, because the staging queue becomes congested more easily.

3 Simulation

We built a simulation of staging queues and flow racks with the simulation package Arena (Kelton et al., 1998). Figure 5 shows simulation and analytical results for a 3-pallet staging queue. We reset the value of λ (and $\mu = 1 - \lambda$) for each replication, varying λ from $0.1, 0.2, \ldots, 0.9$. We cleared the statistics at time 10,000 to remove initial bias and recorded throughput (number of pallets moved) for 100,000 time units.

Figure 6 shows the results for staging queues of different lengths. Maximum throughput is higher for longer queues because arrivals balk less often. Notice that a staging queue is a renewal process that regenerates to state $(0, 0)$. After regeneration, a queue of pallets builds and propagates backward: each arrival moves the rear of the wave one position backward; each service moves the front one position backward. If arrivals and services are approximately balanced, a *pallet wave* forms and "breaks" either at the end of the queue (resulting in a blocked state) or in the middle of its propagation (it "breaks early"). For longer queues, it takes longer for the wave to

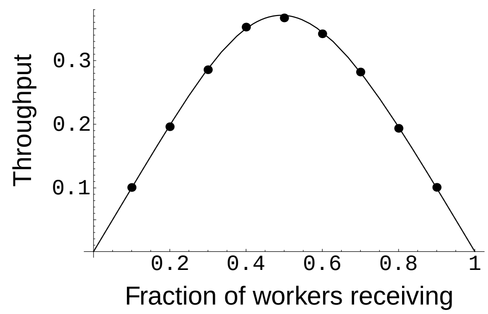

Figure 5: Simulation results for a 3-pallet staging queue, confirming the analytical model. Dots represent results of simulation runs; the solid line represents the analytical result.

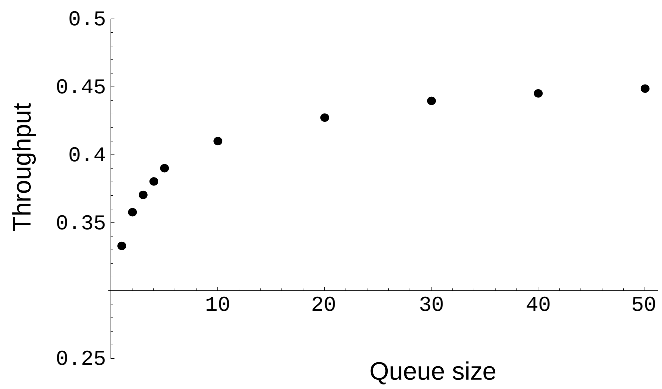

Figure 6: Simulation results for staging queues of different lengths when $\lambda = \mu = 0.5$. Longer queues have higher maximum throughput because they block less frequently. An infinitely long queue never blocks and has throughput 0.5.

reach the blocked state and more waves break early, so the queue is blocked less often.

Our experiments also indicate that for very small and very large sizes, staging queues and flow rack queues perform nearly identically. In the extreme case of size one, both queues are blocked when a single pallet arrives to a busy server; for the infinitely long case, neither queue blocks. Preliminary results suggest that the size at which the two queues differ most is between 10–20 pallets, but that in no case is the difference greater than about 12%. Interestingly, staging queues we have seen in practice are about this size, which is also 1–2 lengths of a trailer.

4 Conclusions

With respect to throughput, staging queues in a crossdock are less efficient than flow rack queues, but only slightly so. In our simulations, throughput for a staging queue was generally within 10% of that for a flow rack queue of the same size. We also observed that longer staging queues accommodate higher throughput, because they are blocked less often.

Our observations suggest that there there is little advantage to using flow racks for crossdocking, especially considering that flow racks have other disadvantages, such as high initial cost and obstruction of material flow patterns.

In the future we intend to address the limitations of the analytical model with the simulation, in particular, we will model a closed system in which workers arriving to a blocked queue wait instead of balk. We also plan to address two-stage crossdocking systems.

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