SPATIAL FILTERING CONSIDERATIONS IN BRAGG DIFFRACTION IMAGING

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ABSTRACT

Many of the techniques and effects observed in images from systems using Bragg diffraction imaging can be explained using concepts of optical spatial filtering. This is possible because the Bragg diffraction phenomenom can be thought of as an interaction of a plane-wave of light with a plane-wave of sound to produce a plane-wave of diffracted light. Changing the incident light causes a change in the diffracted light. Hence, the input light field of the imaging system plays an analogous role to the filter transparency of an optical spatial filtering system. The plane-wave components of the diffracted light that form the image of the sound field can be selectively changed by modification of the incident light field. When the imaging method is analyzed as a plane-wave: plane-wave interaction, simple explanations can be given for such diverse effects as the observed astigmatic resolution, the dependence of the resolution on the semi-apex angle of the incident light wedge, dark field imaging, and reflection imaging.

With the proper choice of the geometric shape and orientation of the input laser beam, we can obtain high-quality optical images of a cross section of the acoustic field from this diffracted light. Different cross sections are obtained by the simple translation of one lens along the axis of the optical system. This ability to obtain all cross sections implies that the acoustic field is reconstructed in its entire volume, a valuable asset in those applications where it may be desirable to scan through the different planes of the volume. This feature is directly analogous to holographic reconstructions.

A diagram of the imaging system is shown in Fig. 2 in a configuration to obtain a transmission-type image. In this arrangement the ultrasonic field insonifying the object is generated by a quartz transducer driven by the rf generator and amplifier. The sound field containing the object information then interacts with the laser light. The laser light is in the form of a wedge with the apex located to the right of the interaction, as in the figure. A vertically oriented cylindrical

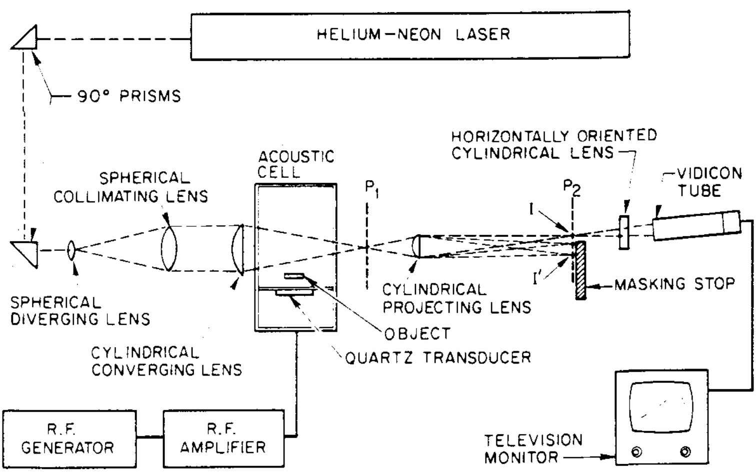
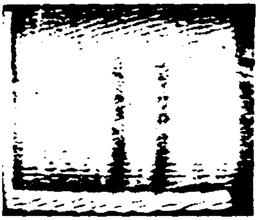


Fig. 2. Schematic diagram of the Bragg imaging system.

INTRODUCTION

Bragg imaging is a method of obtaining a real-time optical image of a cross section of an ultrasonic field. In this method laser light diffracted from a laser beam passing through the sound field forms the image. This acoustic imaging scheme holds promise in the areas of medicine, biology, and nondestructive testing because of its ability to image internal detail in optically opaque material. Figure 1a illustrates the imaging of a biological object - a Silver Dollar fish (*myLossoma argenteum*). Figure 1b is the image of a 7.0-mm thick aluminum plate with two holes (1.5-mm diameter) drilled into the edge with a spacing of 7.5 mm. Internal details are apparent in both of these objects.[1]

(a)



(b)

Fig. 1. Transmission images of (a) a "Silver Dollar" fish, and (b) holes drilled into an aluminum plate.

lens to the left of the acoustic cell acts on a collimated laser beam to form this wedge of light. (The plane of the figure is assumed to be the horizontal plane.) At approximately the plane P_1 two images of the sound field are formed, one to either side of the central order light. The image to the left of the central order light (looking along the axis of light propagation) is an upshifted (in frequency) virtual image of the sound field cross section. The image to the right of the central order component is a downshifted real image. Both images are demagnified in the horizontal direction by the ratio of the light wavelength to the sound wavelength. For rf sound frequencies and He-Ne laser light, this demagnification is of the order of 1/100. The vertical cylindrical projecting lens projects these images to the television camera face, restoring the horizontal dimension of the images to a useful size. At plane P_2 a stop removes the central order beam and one of the images. The remaining image is focused horizontally by the horizontally oriented cylindrical lens. An in-focus real image of the sound field is then picked up directly by a vidicon tube and displayed on a television monitor. Images such as those of Fig. 1 can then be displayed on the screen for real-time viewing or for being photographically recorded.

REVIEW OF BRAGG DIFFRACTION

The primary principle used, Bragg diffraction, is the diffraction of coherent light by a traveling acoustic wave when certain geometric conditions are met.[2,3] In this light-sound interaction the mechanism that diffracts part of the laser beam can be considered as an optical diffraction grating effect caused by a variation in the optical index of refraction due to the presence of the sound traveling through the medium. This effect was first predicted by Brillouin[4] in 1922.

One of the possible approaches to this phenomenon is based upon the interaction of a plane wave of sound with a plane wave of light. When a plane wave of sound intersects a plane wave of light it can be shown that, due to interference effects, the diffracted light wave will have significant amplitude only if the plane waves meet at the proper angle. Representing each wave by its propagation constant, the condition for diffraction may be written vectorially as

$$\vec{k} \pm \vec{K} = \vec{k}_{\pm} \tag{1}$$

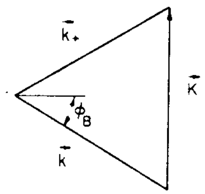
where $\vec{k}$ and $\vec{k}_{\pm}$ are the propagation vectors of the incident and diffracted light, respectively, and $\vec{K}$ is the propagation constant of the sound wave. (It should be noted that here and throughout the analysis to follow we will be concerned only with weak interactions and first-order Bragg diffraction for the intended applications.)

An additional relation can be derived from parametric interaction theory to relate the frequencies of the diffracted waves to the frequencies of the incident waves.[5] This relation is

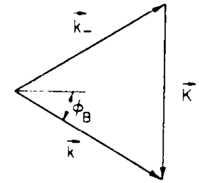
$$\omega_{\pm} = \omega \pm \Omega \tag{2}$$

where ω and $\omega_{\pm}$ are the frequencies of the incident and diffracted light, and Ω is the frequency of the sound. (The $\pm$ relates the frequency of the diffracted wave to the respective sign in the vector equation.) Because of this frequency behavior, the diffracted beams are called the upshifted beam and the downshifted beam. It should be noted from the above frequency relation that the diffracted light frequency is not very different from that of the incident light, since the typical sound frequency used for imaging ($\sim 10^6$ Hz) is very much less than the typical light frequency ($\sim 10^{14}$ Hz). This fact, together with the assumption of a weak interaction, implies that the magnitudes of $\vec{k}$ and $\vec{k}_{\pm}$ will be essentially equal.

The vector relation, Eq. (1), may then be drawn as closed isosceles triangles (called the Bragg triangles), as in Fig. 3.



(a)



(b)

Fig. 3. Bragg triangles for (a) upshifted, and (b) downshifted Bragg diffraction.

From these trianges the required geometric condition for Bragg diffraction may be derived.

$$\sin \phi_B = \frac{|\vec{K}/2|}{|\vec{k}|} = \frac{\lambda}{2\Lambda} \qquad (3)$$

where ϕ_B is the Bragg angle. By drawing the Bragg triangle in a different fashion, as in Fig. 4, the geometric requirements become clearer: the angle between the propagation vectors of the interacting light wave and sound wave must be $\pi/2 \pm \phi_B$ to permit Bragg diffraction.

When the geometrical conditions for diffraction are properly met, the amplitude of the diffracted light wave will be proportional to the product of the amplitudes of the incident sound and light plane waves.[5] The proportionality constant is a function of the interaction medium and the ratio of the acoustic wavelength to the light wavelength. The phase of the diffracted wave is dependent on whether the wave is upshifted or downshifted in frequency. If upshifted, the phase is equal to the sum of the phases of the light and sound plane waves; if downshifted, the phase of the diffracted wave is given by the difference in phase between the incident light wave and the sound wave.[5]

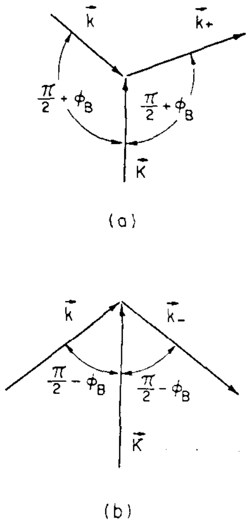


Fig. 4. Propagation vector diagrams for (a) upshifted, and (b) downshifted Bragg diffraction.

The amplitude and phase of the diffracted waves can be represented by complex notation. For example,

$$\underline{U}_+ = |U_+| e^{j\phi_+} \tag{4}$$

where $|U_+|$ is the amplitude and ϕ_+ is the phase of the upshifted plane wave. From the above statements we see that the upshifted and downshifted waves obey the following proportionalities

$$\underline{U}_+ \propto \underline{U}_\ell \underline{U} \tag{5}$$

$$\underline{U}_- \propto \underline{U}_\ell \underline{U}_s^* \tag{6}$$

where $\underline{U}$ and $\underline{U}_s$ are the corresponding complex forms for the incident plane waves of light and sound, respectively, and * represents complex conjugation.

Summarizing the results of the theory of first-order Bragg diffraction from this plane-wave approach:

1. Bragg diffraction can occur only when a plane wave of light and a plane wave of sound intersect so that their propagation vectors have an angular separation given by $\pi/2+\phi_B$ or $\pi/2-\phi_B$ where $\phi_B = \sin^{-1}\lambda/2\Lambda$.

2. The frequency of the diffracted light is shifted by an amount equal to the frequency of the sound where the sign of the shift depends on which angular condition is met.

3. The diffracted light's propagation vector will be directed at an angle of $\pi/2 \pm \phi_B$ from the propagation vector of the sound and will be in the plane determined by the propagation vectors of the input light and sound.

4. The amplitudes and phases of the diffracted waves are specified by the following proportionalities:

$$\underline{U}_+ \propto \underline{U}_\ell \, \underline{U}_s$$

$$\underline{U}_- \propto \underline{U}_\ell \, \underline{U}_s^*$$

where the quantities involved are the complex notation depicting the upshifted and downshifted diffracted plane waves.

SPATIAL FOURIER TRANSFORM REPRESENTATION OF ARBITRARY FIELDS

The utility of the plane-wave approach to Bragg diffraction becomes apparent when combined with the decomposition of an arbitrary sound or light field in terms of its planar-wave components.[6] This is a powerful tool in modern optics and is directly analogous to the Fourier series or transform approach of circuit and systems analysis.

In this method of analysis an arbitrary two-dimensional complex-valued scalar field distribution propagating in the +Z direction can be considered as a sum of plane waves of infinite extent with differing amplitudes and phases. Mathematically we can represent this sum as:

$$\underline{U}(x,y) = \int_{-\infty}^{\infty}\int U'(f_x,f_y)e^{+j2\pi(f_x x+f_y y)}\,df_x df_y \tag{7}$$

where

$$\underline{U}'(f_x,f_y) = \int_{-\infty}^{\infty}\int \underline{U}(x,y)e^{-j2\pi(f_x x+f_y y)}\,dxdy \tag{8}$$

Here each component $\underline{U}'(f_x,f_y)$ can be considered as a plane wave whose amplitude and phase are given by the amplitude and phase of this complex quantity. The propagation direction of this plane wave is simply related to the arguments of $\underline{U}'(f_x, f_y)$. The direction cosines of the propagation direction with respect to the x or y axis are equal to the wavelength times the value of f_x or f_y, respectively. Because of the assumed unidirectional propagation of the field, the direction cosines lie between +90° and -90°. Any values of f_x or f_y greater than λ^{-1} or Λ^{-1} where λ and Λ are the light and sound wavelengths (depending on whether a light field or sound field is being represented) are considered as evanescent waves. Thus the spatial distribution function

$\underline{U}'(f_x,f_y)$ can completely specify any sound or light field.

It should be noted that an equivalent representation of a spatial spectrum can be derived for any coordinate system that can express the propagation direction equally well. One such scheme is that specified by azimuth and inclination angles. Consider, for example, an arbitrary light field $\underline{U}(x,y)$ that propagates in the +z direction. Assuming that the geometric distribution is fairly simple, we can obtain the spatial Fourier transform, $\underline{U}'(f_x, f_y)$ from Eq. (7). Let us now define the inclination angle, β, as the angle that the light component makes with respect to the x-y plane and the azimuth angle, α, as the angle between the +x axis and the projection of the component on the x-y plane. Inclination angles are considered positive above the x-y plane; azimuth angles are positive in the counter-clockwise direction.

To relate this notation to that of the spatial frequencies f_x and f_y, we consider a unit vector of arbitrary inclination and azimuth. We may then represent this vector in Cartesian coordinates as

$$\vec{1} = \hat{a}_z \sin\beta + \hat{a}_x \cos\beta\cos\alpha + \hat{a}_y \sin\alpha\cos\beta \tag{9}$$

The spatial frequencies are specified by relations such as

$$f_x = \frac{(\text{direction cosine})_x}{\lambda} = \frac{\cos\theta_x}{\lambda} \tag{10}$$

where θ_x is the angle between the vector and the x axis.

Thus
$$f_x = \frac{\vec{1}\cdot\hat{a}_x}{\lambda} \tag{11}$$

$$f_x = \frac{\cos\beta\cos\alpha}{\lambda} \tag{12}$$

Similarly,

$$f_y = \frac{\cos\beta\sin\alpha}{\lambda} \tag{13}$$

By substituting these expressions for the arguments of $\underline{U}'(f_x, f_y)$ we can obtain an expression for the spatial distribution function in terms of the angles of inclination and azimuth.

Conceptually the extension of the plane-wave/plane-wave viewpoint of Bragg diffraction to the general interaction of arbitrary light and sound fields is now evident. Simplistically, the approach is as follows: the interacting light and sound fields are each decomposed into plane-wave components. All components which meet the required angular conditions of Bragg diffraction interact to produce a diffracted component. After accounting for diffraction these diffracted components are assembled at any desired plane and the inverse transform performed to find the physical light distribution at that plane. The important contribution to Bragg imaging was Korpel's recognition[7] that the proper choice of the geometric shape of the incident light would cause the diffracted light to reproduce a scaled version of the sound field and hence to produce an image of that sound field.

TWO-DIMENSIONAL INTERACTION FOR IMAGING

To illustrate the simple imaging of a sound field consider a two-dimensional case. We assume that the light source is an infinite vertical line source of unit amplitude and that the sound source is also of infinite vertical extent, but not necessarily a line source (e.g., an infinitely long slit). The magnitudes of the one-dimensional angular spectra for this case are shown in Fig. 5. The spectrum of the line source of light has components of equal magnitude and phase for all angles. The infinite slit spectrum (with respect to the center-line) has a magnitude dependence of the $|\sin\phi/\phi|$ type and a phase variation which changes from 0 to π whenever $\sin\phi/\phi$ becomes negative.

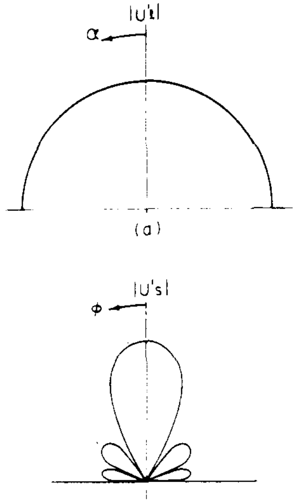


Fig. 5. Angular spectra of (a) an infinite line source of light radiating into only the +x half-space, and (b) an infinite slit source of sound.

The interaction generates a downshifted real image of the sound source in the manner shown in Fig. 6. Consider first the zero-order component of the sound field. Since it must interact at an angle $\pi/2-\phi_B$ we see geometrically that it must interact with the light component propagating at an angle $+\phi_B$ with respect to the x axis. We next consider a sound component at an angle ϕ. In order for it to intersect a light component at an angle $\pi/2-\phi_B$ it must select a light component which propagates at azimuth angle $\alpha = \phi+\phi_B$. The diffracted light component amplitude is directly proportional to the amplitude of the sound component. The phase is the difference of the

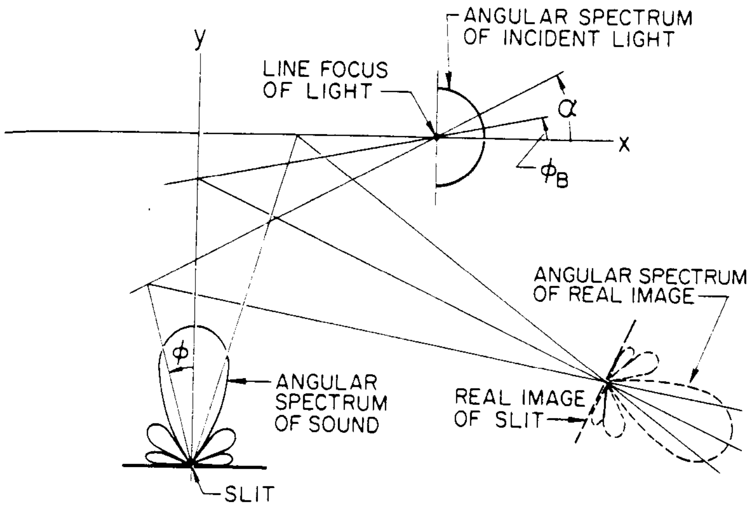


Fig. 6. Two-dimensional interaction to produce a real downshifted image of a slit.

incident light and sound phases. If the phase and amplitude of all incoming light components are equal, as in a line source, then the diffracted light duplicates the sound spectrum but with the propagation reversed.[8] Since all diffracted rays are in the plane of the figure and since the spectrum is reconstructed as the complex conjugate of the sound field, we produce a real image of the sound field at the image plane. Similar analysis of the upshifted case would have produced a virtual image of the field located symmetrically with respect to the undiffracted light beam.

Investigating the diffracted spectrum several observations can be made. First, the optical image is rotated by an angle $\pi/2+\phi_B$ from the acoustic field. Second, the sound spectrum is now reproduced in light. Because of the differences in wavelength this implies a demagnification of the width of the image by the scale of λ/Λ. This demagnification effect is common to all methods where a field of a given wavelength is reconstructed using a second field of a smaller wavelength. These are the imaging rules derived by Korpel from ray tracing arguments in his original paper[7] on Bragg imaging.

Korpel has also derived a relation between the spectra for this two-dimensional case.[9] The relations for the upshifted and downshifted diffracted spectra are, respectively,

$$\underline{U}'_{+}(\psi) = -jc\underline{U}'_{s}(\psi-\phi_B)\underline{U}'_{\ell}(\psi-2\phi_B) \tag{14}$$

$$\underline{U}'_{-}(\psi) = -jc\underline{U}'^{*}_{s}(\psi+\phi_B)\underline{U}'_{\ell}(\psi+2\phi_B) \tag{15}$$

where c is an interaction constant whose value depends on the medium and on the ratio of wavelengths λ/Λ and where * denotes the complex conjugate. The azimuth angles of both the incident and diffracted light spectra are measured with respect to the +x axis. The angle of the sound propagation is measured with respect to the +y axis. These formulae express analytically the heuristic geometrical description given above. From these formulae it is seen that in order to reproduce exactly the sound spectrum in the diffracted light spectrum the incident light spectrum should have constant amplitude and constant phase over all of its angular components. This in turn implies that the best choice of an incident light pattern to image the sound field is a line source. This conclusion was proven later in experimental practice to give the best images of several sources.

SPATIAL FILTERING IN THE TWO-DIMENSIONAL INTERACTION

Because the one-dimensional spectra are multiplied as in Eqs. (14) and (15), it should be possible to form a modified reconstruction of the sound spectrum by tailoring the incident light spectrum. This is similar to the principle used in spatial filtering of optical images.[10] Figure 7 shows one arrangement of optical elements to perform such spatial filtering. The point source S and lens L_1 serve to illuminate an object transparency in plane P_1 with a collimated light beam. Lens L_2 is a Fourier transform lens; the Fourier transform of the object in plane P_1 is formed at plane P_2. This spatial spectrum can then be manipulated by placing a "filtering" transparency in plane P_2 which can attenuate, eliminate, or change the phase of any plane-wave component of the object. This filtering operation is possible because the filter transparency and the Fourier transform light distribution multiply in the plane P_3. Lens L_3 then performs another Fourier transform and the resulting filtered image is displayed (in inverted fashion) in plane P_3. Some examples of the optical operations that can be performed in this fashion are phase contrast, noise removal, matched filtering, correlation, etc.

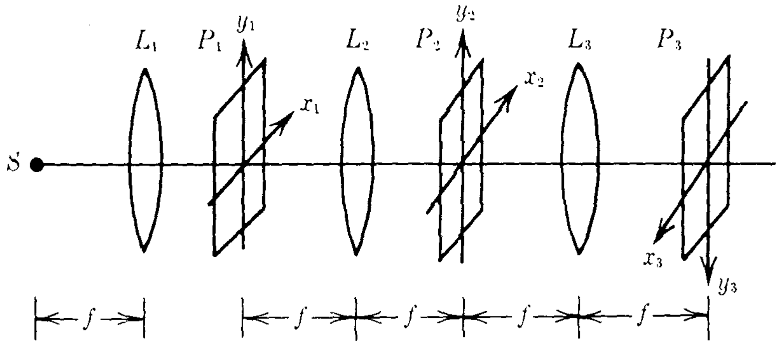


Fig. 7. Schematic diagram of an optical filtering setup.

One of the simplest filter functions is the removal of a component or components from the image spectrum. This is the principle used in contrast reversal (removal of the central order component), removal of periodic noise (removal of spatial component corresponding to periodicity of the noise), edge enhancement (removal of the lower spatial frequencies), etc. The elimination of a particular diffracted light component in the acoustic image can be accomplished by the elimination of the particular incident light component that interacts with the undesired sound component. Two possible methods for the removal of diffracted light components are the use of a modified geometry of the imaging system or the spatial filtering of the incident light.

The first of these methods makes use of the limited angular spectrum of the usual incident light pattern. Physical line sources used in the imaging system have to be introduced through a lens. The semiapex angle of the wedge forming the line image can be taken as a measure of the angular extent of the spectrum. This angle is related to the numerical aperture of the lens (assuming that the lens is fully illuminated) by

$$\alpha_m = \sin^{-1} (\text{N.A.}) \tag{16}$$

where N.A. is the numerical aperture. The angular spectrum of the incident light then can be assumed to have fan-shaped distribution with a sharp cutoff point at $\pm\,\alpha_m$. This distribution is an approximation to the more gradual cutoff of a physical laser beam. It is then possible to make use of this spectral cutoff feature to eliminate all spatial frequencies of the image above a certain frequency by changing the angle of incidence between the sound field and the incident light field.

The validity of this sharp angular spectrum cutoff feature of the incident light in two-dimensional imaging may be checked by measurement of the resolution obtained with different wedge semi-apex angles. In this experiment the central-order propagation components of the incident light and sound are aligned so that they interact to produce the downshifted central-order diffracted component as in Fig. 8a. Since the diffracted

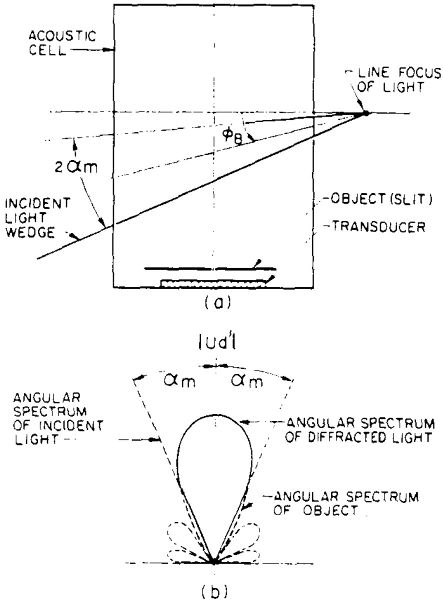


Fig. 8. (a) Physical orientation of the acoustic cell for a normal transmission image. (b) Diffracted light spectrum showing the effect of the limited semi-apex angle of the incident light wedge.

spectrum is proportional to the product of the input spectra, the angular width of the diffracted spectrum is equal to the width of the incident light spectrum as in Fig. 8b (assuming, as is usually the case, that the sound spectrum width is not the limiting factor). As the angular width of the light spectrum is changed, the behavior of the resolution of an acoustic image is an indication of the validity of the assumption of a sharp spectral cutoff of the incident light spectrum since a wider spectrum has better inherent resolution. Based essentially on the same type of analysis Korpel predicted that the image resolution would be given by[7]

$$\xi_h = \frac{\Lambda}{2(\mathrm{N.A.})} \tag{17}$$

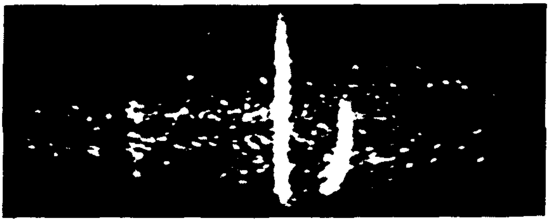
where ξ_h is the minimum resolvable distance in the horizontal direction between two vertical-line sources, and N.A. is the effective numerical aperture of the lens introducing the incident light into the system. Using Eq. (16) this expression becomes

$$\xi_h = \frac{\Lambda}{2\sin\alpha_m} \tag{18}$$

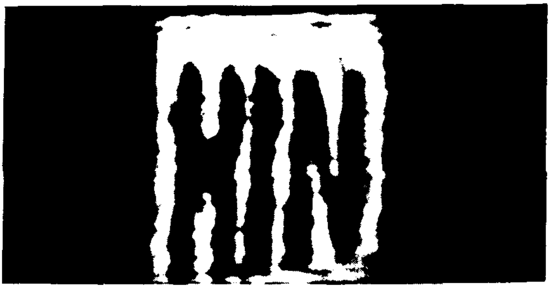
This dependence on the semi-apex angle of the light wedge was confirmed experimentally[11] using long vertical wires as sound scatterers.

Two examples of the spatial filtering technique whereby components are eliminated from the image by using this angular cutoff of the incident light spectrum are "dark field" imaging and acoustic specular reflection imaging. Both of these techniques are characterized by the absence of a diffracted component corresponding to the central order sound component of the illuminating sound from the transducer. Only the sound scattered by or reflected from an object is imaged. Figure 9 shows typical images obtained from each method.[1] There is no image of the transducer sound field present.

To obtain "dark field" imaging it is necessary to rotate the cell through some angle η. The central order component of sound (which is largely the illuminating sound field) will have no component of light with which to interact if the magnitude of η is greater than α_m, the semi-apex angle of the light wedge. The experimental arrangement and the resulting diffraction pattern are sketched in Fig. 10. Note that not only is the central order sound component excluded from the diffracted pattern, but also more of the higher spatial frequencies are included in the image. This method provides images of the sound scattered into higher spatial frequencies from small scatterers. Figure 9a is the "dark field" image of a wire hook.



(a)



(b)

Fig. 9. Images obtained from (a) "dark field" imaging of a wire hook, and (b) specular sound reflection from a glass slide with masking tape letters.

When the cell is rotated to form a "dark field" image and there is no object to scatter sound energy there will be, of course, no image. It is possible, however, to specularly reflect enough sound energy from an object into the incident light field to form a good image of the reflected sound. As shown in Fig. 11, it is possible to orient the reflecting object so that the central order reflected wound component will interact with the central order light component to obtain a standard (i.e., not "dark field" image.)

Fig. 9b shows an image obtained from the sound reflected from a glass slide with masking tape letters which absorb the ultrasound.

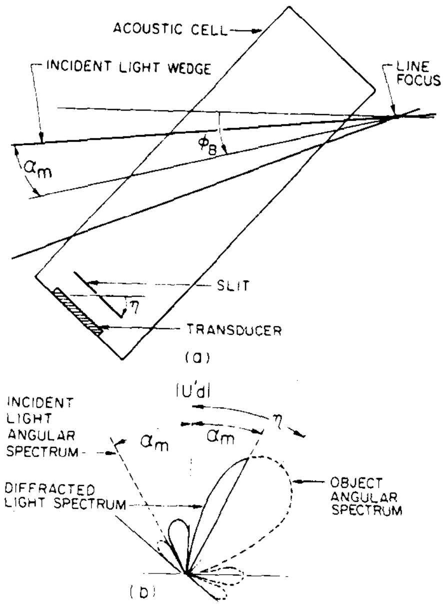


Fig. 10. (a) Physical orientation of the acoustic cell for "dark field" imaging. (b) Angular spectrum of the diffracted light shown as the overlapping portion of the incident light and sound spectra.

The second method of removing an undesired spatial component from the acoustic image is the removal of a particular component from the incident light spectrum by the means of optical spatial filtering. The spatial component that is removed is the component that would interact with the component of sound whose elimination is desired. In a physical imaging system the pseudo-infinite line source of light is formed by focusing a collimated laser beam with a cylindrical lens. The line focus will be located in the back

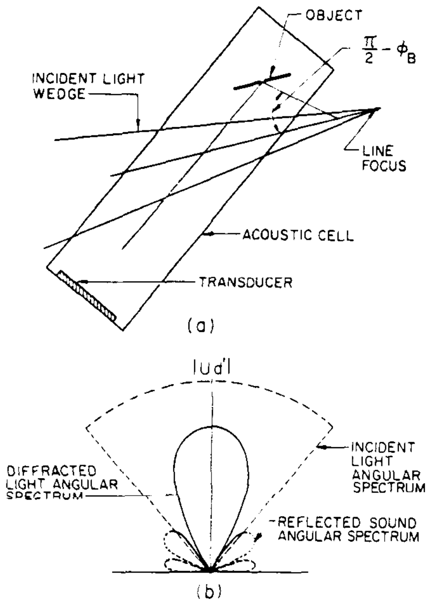
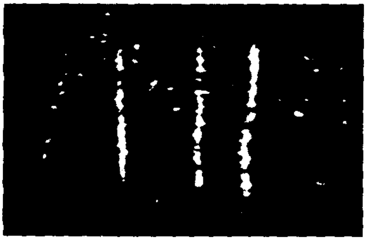


Fig. 11. (a) Physical orientation of the acoustic cell for imaging specularly reflected sound.
(b) Diffracted light spectrum shown as the overlapping portion of the incident light and reflected sound spectra.

focal plane of the lens. The front focal plane of the same lens can serve as the filtering plane for the incident light. In this plane we are free to place one-dimensional filters (long strips of opaque stops) to achieve the desired results. Figure 12 shows the image of three vertical wires before and after spatial filtering of the incident light for contrast reversal. In this method the light component that interacts with the central order component of sound is removed by a stop placed in the filter plane of the incident light. This implies that the diffracted spectrum duplicates all of the sound spectrum except for the missing central order component and thus the contrast of the image is reversed. Similar techniques can be used for other filtering problems.

(a)



(b)

Fig. 12. Acoustic images of vertical wires (a) with normal transmission method, and (b) with spatial filtering of the incident light to achieve contrast reversal.

It should be noted that this method has greater flexibility than that previously described since attentuating and phase shifting filters can be used, as well as stops. The first technique of rotating the cell has only limited applications where it might be desirable to eliminate all spatial frequencies above or below a certain cutoff. Both methods are limited to the two-dimensional interaction, or equivalently, long vertical objects in the sound field.

THREE-DIMENSIONAL INTERACTION

The previous two-dimensional analysis predicted images of infinite vertical objects with good horizontal resolution. It is now of interest to consider the interaction for horizontal objects with vertical detail. To investigate this case we will assume that the sound source is infinitely long in the horizontal direction and that the incident light still forms an infinite vertical line source. These assumptions imply that all light components will be horizontal and all sound components will lie in the vertical plane.

As shown in Fig. 13, we represent the azimuth angle (with respect to the +x axis) of a general light component as the angle, α. The inclination of a sound component from the x-y plane is given by angle, θ. We now seek a relation between these angles that will hold when the components intersect at an angle $\pi/2-\phi_B$, insuring a Bragg interaction. A detailed view of the interacting

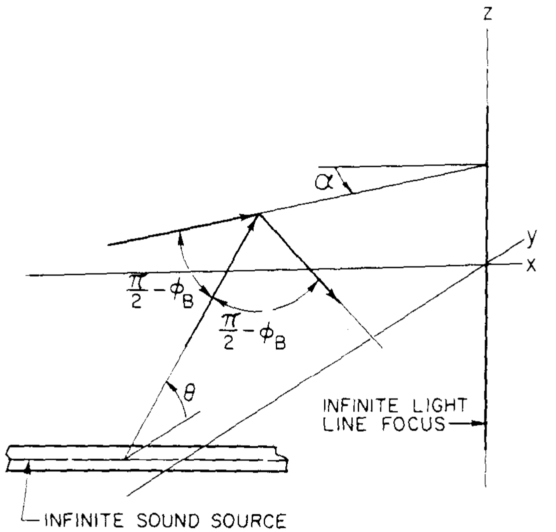


Fig. 13. Geometry for three-dimensional interaction involving infinite sources.

components relative to the intersection point is shown in Fig. 14. Applying Napier's rule to the right spherical triangle shown,[11] we obtain the required relation between the propagation directions of light and sound components

$$\sin\phi_B = \sin\alpha\cos\theta \tag{19}$$

From this type of relation one can predict the light component, specified by α, that would interact with any given sound component, specified by θ.

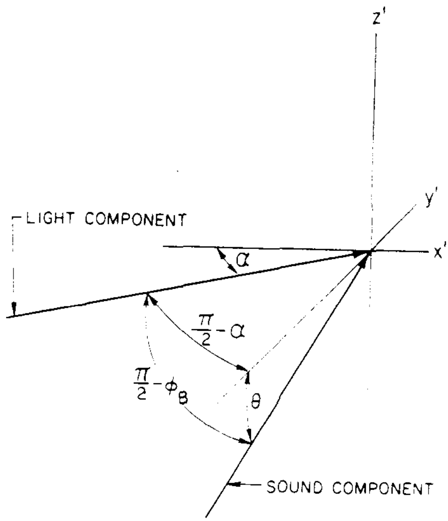


Fig. 14. Detailed view of the interacting components from infinite sources.

One useful fact derived from this relation is that the size of the angular wedge of light determines the resolution in the vertical dimension. Since the input wedge of light can be assumed to have an angular spectrum cutoff at some angle α_m (usually just the semi-apex angle of the wedge), there is a related maximum value of sound component inclination, θ_m. Since no sound components of greater inclination will be imaged, it is this maximum value of inclination that determines the resolution of vertical detail, according to the relation

$$\xi_v = \frac{\Lambda}{2\sin\theta_m} \tag{20}$$

where ξ_v is the minimum resolvable detail in the vertical direction. Experimental results[11] confirm this expression when the height of the line source can be considered infinite. In some experimental situations the physical height of the noninfinite light wedge can be the limiting factor of resolution since it has an effect similar to the pupil function of a lens, limiting the spatial frequencies of the image to those spatial components intersecting the interaction region. When compared with the vertically oriented sound source, it has been found that the size of the resolution element for horizontal objects is significantly less (2/3 of an acoustic wavelength to 10 acoustic wavelengths) than that for vertically oriented objects.

We now wish to consider the general case of interaction between arbitrary components of light and sound. We seek a relation that would identify all light components that could interact with a given specific sound component. Since the angular condition that components intersect at $\pi/2\pm\phi_B$ must be satisfied, this implies that all light components that lie in a cone whose axis is the given sound component and whose semi-apex angle is $\pi/2\pm\phi_B$ can interact with that sound component. Such a situation is illustrated in Fig. 15. In some special cases geometrical considerations will reduce the number of light components that can interact. In the simplest cases (such as those discussed previously) only one component of light can interact with one component of sound because of the planar nature of the light spectrum. This one-to-one correspondence is a desirable feature for imaging since there would be no redundant information in the diffracted light. Of course it is also required that the aberrations of the image not be too severe when introduced by such factors as nonfocussed diffracted components or phase differences due to differing path lengths for various components.

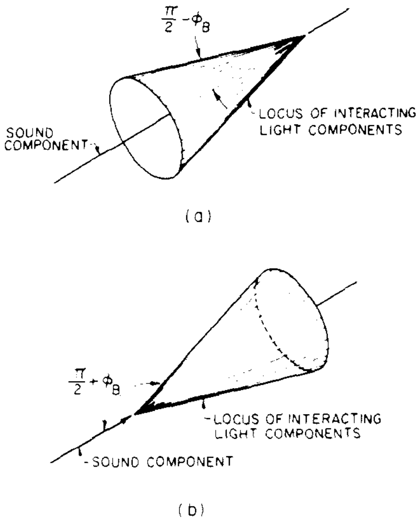
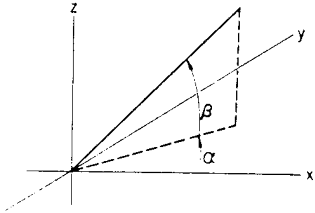


Fig. 15. Angular condition showing all possible interacting light components for a given acoustic propagation vector for (a) downshifted diffraction and (b) upshifted diffraction.

For the purpose of specifying a component we again use an azimuth angle and an inclination angle coordinate system. Figures 16a and 16b show the coordinate system for the incident light and the sound, respectively. For the incident light distribution the inclination angle β measures the angle that the component makes with respect to the x-y plane; the azimuth angle, α, is the angle between the +x axis and the projection of the component on the x-y plane. Similarly the angle θ measures the inclination of the sound component with respect to the x-y plane; the aximuth angle ϕ is the angle between the +y axis and the projection of the sound component on the x-y plane.



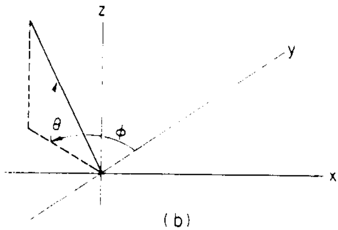


Fig. 16. Angular coordinates for (a) the incident light, and (b) the sound field.

Inclination angles are considered positive above the x-y plane; azimuth angles are positive in the counterclockwise direction.

To derive the relation between the components that can interact we assume that a sound component at an inclination θ and an azimuth ϕ is interacting with a light component at inclination β and azimuth α. As in Fig. 17 these components must intersect an an angle of $\pi/2-\phi_B$ in order to interact. (A similar analysis holds for the upshifted case where the angle of intersection is $\pi/2+\phi_B$.) Figure 18 shows the interaction point with the angular quantities noted. In order to obtain a relation between the quantities we must relate the sides of the sperical quadrilateral formed by the angular arcs (shown in Fig. 18 as BCDE).

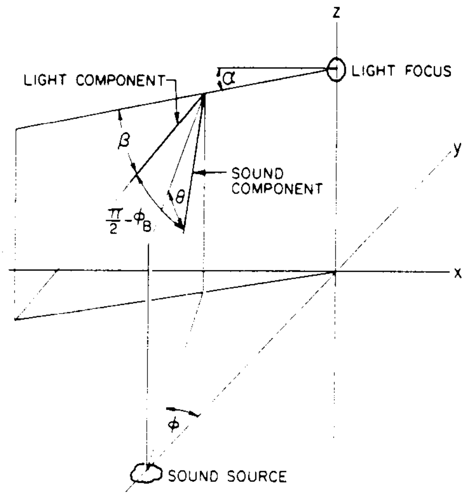


Fig. 17. Interaction between arbitrary sound and light components.

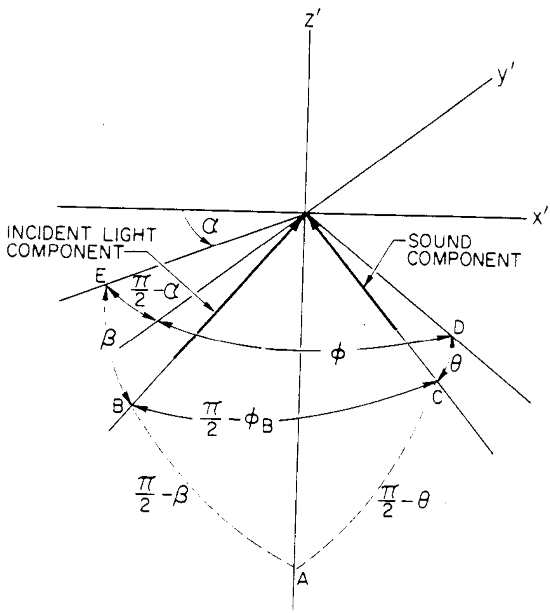


Fig. 18. Detail view of component intersection point.

Because of the geometry of the quadrilateral it is possible to find any one side in terms of the other three. One useful solution is to find β in terms of the other quantities. This is equivalent to determining the interacting ray's inclination while assuming knowledge of its azimuth and the interacting sound ray's azimuth and inclination.

To solve for β we first construct the spherical triangle ABC of Fig. 18 by continuing the meridian lines specifying β and θ to the pole of the hypothetical sphere on which the arcs are drawn. The arcs AB and AC will be complementary to β and θ. The angle A is equal to the angle measured along the equator, i.e., $\pi/2-(\alpha-\phi)$. Solving this oblique spherical triangle for β in terms of the other quantities we find

$$\beta = \pi/2-\cos^{-1}\left[\frac{1}{\sqrt{1+\dfrac{\sin^2(\alpha-\phi)}{\tan^2\theta}}}\right]$$

$$-\cos^{-1}\left[\frac{1}{\sqrt{1+\dfrac{\sin^2(\alpha-\phi)}{\tan^2\theta}}}\cdot\frac{\sin\phi_B}{\sin\theta}\right] \quad (21)$$

This is the desired relation giving the inclination of the light component in terms of its azimuth and the angular coordinates of the given sound component. As noted previously, the amplitude and phase of the diffracted wave are determined respectively by the product of the amplitudes and the difference of the phases of the interacting components specified by this equation.

From this equation we note that for a specified sound component α serves as a parameter in determining the inclination angle β. Thus as α varies, the value of β will also change indicating interaction unless the imaged object approximates an infinitely long vertical object. If not, removal or modification of a light component which interacts with the undesired sound component now implies that other spatial components of the image will be removed. While it is possible to predict which components will be affected (by Eq. (23)) this feature is generally undesirable for spatial filtering where a one-to-one correspondence is desired for precise control of the filtered image. Of course it is still possible to spatially filter the optical image of the acoustic field by conventional optical techniques before viewing the filtered image.

More information regarding verification of Eq. (21) can be obtained by making assumptions about the size and orientation of the sound source. In particular it is instructive to consider an infinite sound source oriented either vertically or horizontally. When the infinite sound source is vertical, then the spectrum varies only in azimuth and we can assume $\theta = 0$. Equation (22) then reduces to

$$\sin(\alpha-\phi) = \sin\phi_B \tag{24}$$

or

$$\alpha = \phi_B+\phi \tag{25}$$

Referring to the *geometric two-dimensional* analysis we see that this expression is the same as that previously obtained. This is expected since the case of two infinite parallel sources is exactly the same as the two-dimensional case.

When the infinite sound source is oriented horizontally, the sound spectrum varies only in inclination, and thus $\phi = 0$. Then Eq. (22) becomes

$$\sin\alpha \cos\theta = \sin\phi_B \tag{26}$$

that several light components will interact with the given sound component. This interaction with several light rays is undesirable for imaging because of the resulting ambiguity of the information. Thus we conclude that for imaging purposes the best light source is the infinite light source or an approximation since this restricts the spectrum to be one dimensional. With this type of source the components are restricted to being coplanar and the ambiguity of information is removed. Aberrations may still be present, however, that limit the usefulness of the image.

IMAGING WITH INFINITE LINE SOURCES OF LIGHT

With the assumption that an infinite light source produces an unambiguous image (although not necessarily without aberration) it is instructive to consider some special cases involving various orientations of the infinite light source. When the infinite light source is oriented vertically the expression of Eq. (21) simplifies to

$$\sin(\alpha-\phi)\cos\theta = \sin\phi_B \tag{22}$$

$$\alpha = \sin^{-1}\left[\frac{\sin\phi_B}{\cos\theta}\right] + \phi \tag{23}$$

This is a particularly important configuration since current Bragg imaging systems use vertical line sources or images (although not infinite) to obtain the best images of arbitrary sound fields. From Eq. (23) it is seen that a given component of the planar light spectrum specified by the azimuth angle α interacts with an infinity of sound components whose angular coordinates satisfy the above relation. Here it is implicitly assumed that the input laser beam is intense enough to interact with all possible sound components. Because of this one-to-many interaction for a general sound field the possibility is reduced for spatially filtering the image by modifying the input light beam as in the two-dimensional

This case is the same as that used to introduce the three-dimensional interaction, and the relation of Eq. (26) is the same as that derived geometrically (Eq. 19)).

Another case of interest is the interaction of a general sound field with the light from an infinite horizontal light source. For this imaging system the light spectrum varies only in inclination but not in azimuth ($\alpha=0$). For the special case of the infinite horizontal light source, Eq. (21) reduces to

$$\beta = \frac{\pi}{2} - \cos^{-1}\left[\frac{1}{\sqrt{1+\frac{\sin^2(-\phi)}{\tan^2\theta}}}\right]$$

$$-\cos^{-1}\left[\frac{1}{\sqrt{1+\frac{\sin^2(-\phi)}{\tan^2\theta}}}\cdot\frac{\sin\phi_B}{\sin\theta}\right] \quad (27)$$

Again the one light component β can interact with all sound components whose angular coordinates satisfy the above relation.

When further restrictions are made on the sound source limiting it to an infinite source, further simplifications of the relation occur. When the infinite sound source is vertically oriented ($\theta=0$) the relation becomes

$$\beta = \cos^{-1}\left[\frac{\sin\phi_B}{\sin(-\phi)}\right] \quad (28)$$

When the infinite sound source is oriented horizontally ($\phi=0$), the inclination of the interacting light ray is given by

$$\beta = \sin^{-1} \frac{\sin\phi_B}{\sin\theta} \qquad (29)$$

From these relations we have verified that the choice of a line source of light is the best when considered on the basis of avoiding a redundancy of information in the diffracted light. This choice has been derived without consideration of possible aberrations in the image which depends in part on the orientation of the line source. Preliminary analyses[8] of some cases involving infinite sound sources show that two vertical infinite sources will give an essentially undistorted image while a vertical light source and a horizontal sound source give an aberrated image. Consideration of both facets of the imaging problem would be necessary to determine the light distribution and orientation that would give the best image for any general three-dimensional sound field.

SUMMARY

Consideration of Bragg diffraction imaging using a plane-wave analysis yields much information about this imaging technique. As might be expected the two-dimensional analysis gives, in a fairly simple fashion, a great deal of information about the formation of images and such diverse effects as the observed resolution, dark field imaging and novel spatial filtering techniques of the acoustic image. These results are based on the present Bragg diffraction imaging systems that use a quasi-infinite vertical line image of incident light. The geometrically more complicated three-dimensional analysis gives less quantitative information about the image. Angular relations between the interacting light and sound components have been derived. From this relation some simple qualitative predictions about image quality can be made.

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