TRANSIENT FIELDS FROM FOCUSED ACOUSTIC WAVES

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ABSTRACT

Calculation of the resolution of an imaging system or of the expected scattered fields requires knowledge of the insonifying field. This paper presents a method of calculating the field of a focused wave from a source with arbitrary wave curvature and arbitrary time excitation. The method can be used for calculating the field at any plane in front of an arbitrary curved wavefront of finite extent. The region surrounding the source is assumed to be rigid baffle (although the rigid-baffle results can be extended to include other baffle conditions). The technique finds the spatial impulse response (i.e., the response to the source when excited by a temporal impulse function) for the arbitrary spatial shape of the wave. The solution for arbitrary time excitations is easily found by a time convolution with this spatial impulse response. The method is demonstrated for propagation in lossless media, but can also be extended to lossy media as the propagation process is modelled by a multiplicative spatial filter, that is changed according to the assumed medium and boundary conditions. Monotonic focused sources with an impulse time excitation can be interpreted as a time-varying line source with decreasing radius. The solution of the wave field is a superposition of the field from the present line source and the fields from all sources in the past. The resulting plots allow evaluation of the resolution capabilities of the field, the sidelobe structure, and wave behavior both in front of and behind the plane of best focus. Examples offered include a spherical concave wave, a conical wave, a parabolic wave, and a spherical convex wave.

THEORY

This paper describes a technique for the efficient computer computation of the transient field of a curved wave front. This wave front can be produced from a focused transducer, from a planar array with proper phasing to produce the curved wave, or from a wave that has transited an acoustic lens. It is assumed that the curved wave passes through an aperture in a rigid baffle and that the medium is linear, homogeneous, and lossless.

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For a planar rigid-baffled transducer, it is known from diffraction theory that the velocity potential is related to the source velocity distribution by

$$\phi(x,y,z,t) = T(t) \underset{t}{*} s(x,y) \underset{xy}{**} [\delta(ct-R)/2\pi R] \tag{1}$$

where $s(x,y)$ and $T(t)$ represent the space and time-varying parts of the known separable velocity disturbance at the input plane, $R=(x^2+y^2+z^2)^{1/2}$ and * indicates convolution performed over the indicated variable. The term to the right of the time convolution is the 'spatial impulse response'[1] of the propagation $h(x,y,z,t)$. The curvature of the wave can be modelled by a spatially variable delay[2] $d(x,y)$ from a plane wave.

$$h(x,y,z,t) = s(x,y)\delta[ct-d(x,y)] \underset{xyt}{***} \delta(ct-R)/2\pi \tag{2}$$

where $d(x,y)$ is the spatial offset of the curved wave front from a plane wave. All one needs for the propagation technique to be described is a description of the wave front in terms of its relative displacement, $d(x,y)$.

Because of the difficulty of the spatial convolutions in Eq. 2, it is convenient to use the spatial frequency domain. Propagation in this domain corresponds to a time generalization of the angular spectrum theory, leading to a linear systems interpretation of the transient diffraction. Reference 3 shows that the spatial transform of the spatial impulse response is given by (neglecting multiplicative constants,

$$H(f_x,f_y,z,t) = B[s(r)\delta[ct-d(r)] \underset{t}{*} J_0[\rho(c^2t^2-z^2)^{1/2}]H(ct-z) \tag{3}$$

where $B[\cdot]$ is the Hankel transform operator, and $\rho=(f_x^2+f_y^2)^{1/2}$.

The transform on the right side of Eq. 3 can be evaluated[3] as

$$B[s(r)\delta(ct-d(r)] = \sum_{i=1}^{N} \frac{s(r_i^*)J_0(\ r_i^*)r_i^*}{\left|\frac{\partial d(r)}{\partial r}\right|_{r=r_i^*}} \underset{t}{*} J_0[\rho(c^2t^2-z^2)^{1/2} \tag{4}$$

where $r_i^*(y)$ represents the values of r for which $d(r)-ct=0$. Here, N is the number of r_i^*.

The temporal impulse response $h(x,y,z,t)$ is obtained by inverse transforming Eq. 4 to give [2],

$$h(x,y,z,t) = B^{-1}\left|\sum_{i=1}^{N} \frac{s(r_i^*)J_0(\rho r_i^*)r_i^*}{\left|\frac{\partial d(r)}{\partial r}\right|_{r=r_i^*}} \underset{t}{*} J_0[\rho(c^2t^2-z^2)^{1/2}\right| \tag{5}$$

This equation gives the output field for an axially symmetric surface velocity when excited by an impulse in time. The curvature of the field is contained in the expressions for r_i^*. (Reference 3 contains equations that are valid for all cases, not just the axisymmetric. Reference 4 considers the effects of the assumed boundary conditions and Ref. 5 considers propagation in lossy media.)

The terms of the summation represent the angular spectrum of a circular line source weighted by the function, $s(r_i^*)$. The radius of such a line varies with time according to the delay law since r_i^* is a function of time. The resulting field is just the summation of these line-generated waves plus the diffraction field that has been generated by the previous line excitations. The field computation requires only one convolution for the each spatial frequency and one Hankel transform.

NUMERICAL SIMULATIONS

Once the geometry of the transducer is known, an elementary calculation leads to the relative displacement of the wave, d(r). Then the N zeros of ct-d(r) are calculated. These are $r_i^*(y)$. These solutions are then used in Eq. 5. Standard algorithms perform the transforms. (It is worth noting that the calculation of the convolution uses the same products required for the transform, thereby reducing the computational complexity of the required operations.)

If the displacement, d(r), is a monotonic function, then the intersection with the plane, z=ct, will reduce to a closed line and the summation in the equations will reduce to a single term. This is usually the case for a focused wave. The following simulations have been investigated using this technique: circular wave fronts with concave curvatures of spherical, conical and parabolic shapes.

The computations were done on a grid of 64x64 spatial sample points and 50 time samples. The plots show one spatial dimension vs. time for a median through the center of the transducer. The complete three-dimensional calculation consumes approximately 80 seconds of CPU computer time on an IBM 3033 mainframe computer. For convenience the plots have been normalized to a maximum value of one. The time axis as well as the width axis are expressed in terms of one characteristic size, A, of the transducer (either half-width or radius, as appropriate). The time axis has a zero value at the time when the first wave reaches the observation line. The focal length f is 10 cm in all cases. The transducer radius, A, is assumed to be 2.0 cm.

Figures 1 and 2 show examples of the diffraction pattern at the focal plane for spherical and conical waves. While the solution of Eq. 5 is obtained in a plane parallel to the excited surface the plots have been shown with time as the variable to allow comparison with existing solutions. Similar plots were obtained for conical waves. Additionally it is possible[2] to predict the boundaries of significant wave energy if one wishes only to know the region of the wave without knowing the details of the wave amplitude spatial dependence.

ACKNOWLEDGEMENTS

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REFERENCES

1. See, for example, P.R. Stepanishen, "Experimental verification of the impulse response method to evaluate transient acoustic fields", J. Acoust. Soc. Am. **63**(6), pp. 1610-1617, 1981

2. H. Lasota, R. Salamon, and B. Delannoy, "Acoustic diffraction analysis by the impulse response method: A line impulse response approach", J. Acous. Soc. Am., **76**(1), pp. 280-290, 1984

3. D. Guyomar and J. Powers, "Transient fields radiated by curved surfaces- application to focusing", J. Acoust. Soc. Am., **76**(5), pp. 1564-1572, 1984

4. D. Guyomar and J. Powers, "Boundary effects on transient radiation fields from vibrating surfaces", J. Acous. Soc. Am., 77(3), pp. 907-915, 1985

5. D. Guyomar and J. Powers, "Propagation of transient acoustic waves in lossy and lossless media", this volume

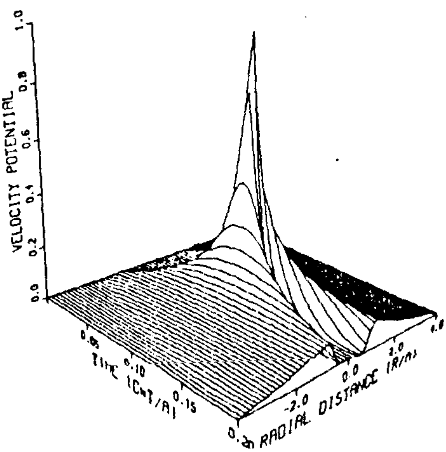

Figure 1 Spherical concave wave with a circular cross-section
(Impulse excitation, A=2.0 cm, f= 10 cm, z=10 cm)

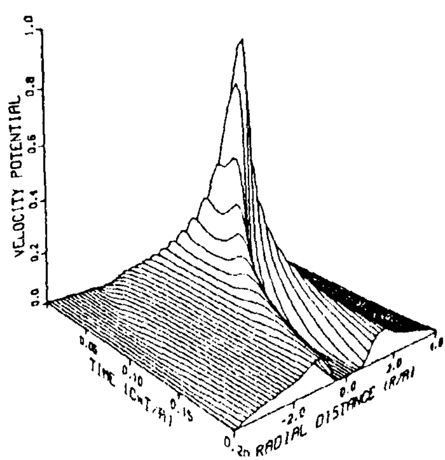

Figure 2 Conical concave wave with a circular cross-section
(Impulse excitation, A=2.0 cm, f= 10 cm, z=10 cm)