EC 4210 Solutions

Assignment 7

19.3. Consider a light bulb with an equivalent temperature of 2800 K.

a. Find fraction of the optical power output that lies in the visible region of the spectrum (between 300 and 700 nm).

b. Find the fraction of the optical power output that lies between 1 μm and 10 μm.

Solution: a. Using Figure 19.4 on page 283 (or a blackbody slide rule), we find that ...

- ... $\approx 0.004\%$ of the power lies below 300 nm.
- ... $\approx 6.2\%$ of the power lies below 700 nm.

So $(6.2 - 0.004) \approx 6.2\%$ of the power lies in the visible part of the spectrum.

b. Using Figure 19.4 on page 283 (or a blackbody slide rule), we find that **Revised 3/2/99**

- ... $\approx 23\%$ of the power lies below 1 μm.
- ... $\approx 100\%$ of the power lies below 10 μm.

So $(100 - 23) = 77\%$ of the power lies in this part of the infrared spectrum.

We note that considerably more IR power is generated than visible light, lowering the light bulb's efficiency (hence, the invention of fluorescent lights with their higher conversion efficiency).

19.4. Consider a HeNe laser operating at 632.8 nm with a linewidth of 10^6 Hz that produces 1 mW of output power in a beam of radius 1 mm in size.

a. Compute the number of photons/second produced by this laser.

b. Compute the equivalent temperature that a blackbody source would have to produce the same spectral radiant photon emittance as the laser.

Solution: a. The number of photons per second, N', produced by this laser is

$$N' = \frac{P_{\text{out}}}{h\nu} = \frac{P_{\text{out}}\lambda}{hc} = \frac{(1 \times 10^{-3})(632.8 \times 10^{-9})}{(6.63 \times 10^{-34})(3.0 \times 10^8)} = 3.18 \times 10^{15} \text{ photons} \cdot \text{s}^{-1} \,. \tag{1}$$

b. The effective emitting area of the laser is

$$A = \pi r^2 = \pi(1 \times 10^{-3})^2 = 3.14 \times 10^{-6} \text{ m}^2 \,. \tag{2}$$

The spectral linewidth $\Delta\lambda$ of the source is

$$\Delta\lambda = \frac{\lambda^2 \Delta\nu}{c} = \frac{(632.8 \times 10^{-9})^2 (1 \times 10^6)}{3.0 \times 10^8} = 1.335 \times 10^{-15} \text{ m}. \tag{3}$$

c. We know that for a blackbody $Q_\lambda \Delta\lambda A =$ the number of photons emitted per second into a spectral width $\Delta\lambda$. We can calculate an equivalent Q_λ for the laser of $Q_\lambda \Delta\lambda A = N'$, so

$$Q_\lambda = \frac{N'}{\Delta\lambda A} = \frac{3.18 \times 10^{15}}{(1.335 \times 10^{-15})(3.14 \times 10^{-6})} = 7.50 \times 10^{33}. \tag{4}$$

We also have the formula,

$$\begin{aligned}
Q_\lambda &= \frac{2\pi c}{\lambda^4} \frac{1}{e^{\frac{hc}{\lambda kT}} - 1} \qquad (5) \\
7.50 \times 10^{33} &= \frac{2\pi c}{\lambda^4} \frac{1}{e^{\frac{hc}{\lambda kT}} - 1} \\
e^{\frac{hc}{\lambda kT}} - 1 &= 1.550 \times 10^{-2} \\
e^{\frac{hc}{\lambda kT}} &= 1.015 \\
\frac{hc}{\lambda kT} &= 1.538 \times 10^{-2} \\
T &= \frac{hc}{\lambda k (1.538 \times 10^{-2})} \\
&= \frac{6.63 \times 10^{-34} 3.0 \times 10^8}{(632.8 \times 10^{-9})(1.38 \times 10^{-23})(1.538 \times 10^{-2})} \\
&= 1.479 \times 10^6 \text{ K}.
\end{aligned}$$

19.5. Consider the sun as a blackbody source with an equivalent temperature of 6000 K.

a. Find λ_m and W_λ at λ_m.

b. Find the total radiant power emitted by the sun if its approximate diameter is 1.39×10^6 km.

c. Find the solar irradiance at the earth's orbit if the average earth-sun distance is 149×10^6 km.

d. Compute the power intercepted by the earth if the average earth diameter is 12,750 km. (The earth can be modeled as a flat disk of this size.)

e. If two-thirds of the irradiance of part c is transmitted through the atmosphere, calculate the size of a square solar panel array that is required to generate 1 kW·hr of energy per day. Assume that the conversion efficiency of the array is 10% and that the average day of sunshine is 8 hours long.

Solution: a. We want to find λ_m and $W_\lambda(\lambda_m)$. We know that $\lambda_m T = 2898$, so

$$\lambda_m = \frac{2898}{T} = \frac{2898}{6000} = 483 \text{ nm}, \tag{6}$$

and

$$W_\lambda(\lambda_m) = bT^5 = (1.28 \times 10^{-15})(6000)^5 = 9.95 \times 10^3 \text{ W} \cdot \text{cm}^{-2} \cdot \mu\text{m}. \tag{7}$$

b. The surface area of the sun is

$$A = \frac{4\pi d^2}{4} = \pi d^2 = \pi(1.39 \times 10^{11})^2 = 6.07 \times 10^{22}\ \text{cm}^2\,. \tag{8}$$

The total power P is

$$P = WA = \sigma T^4 A = (5.67 \times 10^{-12})(6000)^4(6.07 \times 10^{22}) = 4.46 \times 10^{26}\ \text{W}\,. \tag{9}$$

c. The irradiance is

$$H = \frac{P}{4\pi R^2} = \frac{4.46 \times 10^{26}}{4\pi(1.49 \times 10^{13})^2} = 1.59 \times 10^{-1}\ \text{W} \cdot \text{cm}^{-2}\,. \tag{10}$$

d. Modeling the earth as a flat disk of diameter $d = 1.27 \times 10^9$ cm, we can estimate the solar power P_2 intercepted by the earth as

$$P_2 = HA_{\text{earth}} = \frac{H\pi d^2}{4} = \frac{(1.59 \times 10^{-1})(\pi)(1.27 \times 10^9)}{4} = 2.04 \times 10^{17}\ \text{W}\,. \tag{11}$$

e. If 67% of the the solar irradiance reaches the surface of the earth (due to atmospheric absorption), we have that

$$H_{\text{surface}} = 0.67(1.59 \times 10^{-1}) = 1.065 \times 10^{-1}\,. \tag{12}$$

We know that 1 kW-hr is the energy requirement and that we have to accumulate this energy in a time period $T = 8$ hrs. If the conversion were perfect, we need to collect a power of

$$P = \frac{U}{T} = \frac{1}{8} = 0.125\ \text{kW}\,. \tag{13}$$

Because our collection and conversion efficiency is only 10%, we actually need a power of

$$P_2 = \frac{P}{\text{Eff}} = \frac{0.125}{0.1} = 1.25\ \text{kW}\,. \tag{14}$$

To generate 1.25 kW we want an area of

$$A = \frac{P_2}{H_{\text{surface}}} = \frac{1.25 \times 10^3}{1.065 \times 10^{-1}} = 1.174 \times 10^4\ \text{cm}^2\,. \tag{15}$$

So, a square solar collector that is 1.083×10^2 on a side (or approximately 1 m x 1 m) is required.

20.2. Consider a medical tumor detection system that looks for a contrast between the warm tumor and its slightly cooler surroundings. Calculate the best wavelength of operation for the system.

Solution: To maximize the *contrast* at a body temperature of 310K, we want $\lambda_c T = 2411$, so

$$\lambda_c = \frac{2411}{T} = \frac{2411}{310} = 7.77\ \mu\text{m}\,. \tag{16}$$

20.3. A helicopter design team has budgeted a power of 50 watts to operate a blackbody source as an IR beacon that can be seen with night vision goggles at 1.0 μm.

a. What is the optimum operating temperature for this beacon?

b. What fraction of the output optical power will fall in the visible spectrum?

c. ... of its photon output?

Solution: a. This is an engineering efficiency problem, so we want $\lambda T_e = 3625$ and, so,

$$T_e = \frac{3625}{\lambda} = \frac{3625}{1} = 3625 \text{ K}. \tag{17}$$

Thus, we want the blackbody to operate at a temperature of 3,625K.
b. Using Figure 19.4 on page 283 (or a blackbody slide rule), we find that ...

- ... $\approx 0.1\%$ of the power lies below 300 nm ($\lambda T = 1087\ \mu\text{m} \cdot \text{K}$).
- ... $\approx 17.8\%$ of the power lies below 700 nm ($\lambda T = 2538\ \mu\text{m} \cdot \text{K}$).

So $(18 - 0.1) \approx 17.9\%$ of the power lies in the visible part of the spectrum.
c. Using Figure 19.8 on page 289, we find that ...

- ... $\approx 2\%$ of the power lies below 300 nm ($\lambda T = 1087\ \mu\text{m} \cdot \text{K}$).
- ... $\approx 8\%$ of the power lies below 700 nm ($\lambda T = 2538\ \mu\text{m} \cdot \text{K}$).

So $(8.0 - 2.0) \approx 6.0\%$ of the photon emittance lies in the visible part of the spectrum.