EC 4210 Solutions

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Assignment 6

17.4. Consider the photodiode used in direct detection applications with the following specifications:

Detector	Signal
$\eta = 50\%$	$\lambda = 920$ nm
$i_d = 1$ pA	$m = 1$
$R_L = 500\Omega$	$B = 100$ MHz

a. Calculate the signal power that would be required to have the signal-dependent shot noise dominate (by a factor of ten) the dark current-dependent shot noise.

b. To dominate the thermal noise?

c. Find the minimum detectable power for this detector if thermal noise is dominant.

Solution: a. We want the shot noise due to the signal to be ten times the shot noise due to the dark current, so

$$2q\bar{i}B = 10(2q\overline{i_d}B)), \tag{1a}$$

and, so,

$$\bar{i} = 10\overline{i_d}. \tag{1b}$$

Hence,

$$\frac{P_s q\eta\lambda}{hc}\left(1+\frac{m^2}{2}\right) = 10\overline{i_d}, \tag{2a}$$

and

$$\begin{aligned} P_s &= \frac{hc(10\overline{i_d})}{q\eta\lambda\left(1+\frac{m^2}{2}\right)} = \frac{(6.63\times10^{-34})(3.0\times10^{8})(10)(1\times10^{-12})}{(1.6\times10^{-19})(0.50)(920\times10^{-9})\left(1+\frac{(1)^2}{2}\right)} \\ &= 1.804\times10^{-11} = 18.04 \text{ pW}. \end{aligned} \tag{2b}$$

b. We now want the shot noise due to the signal power to be ten times the thermal noise.

$$2q\frac{P_s q\eta\lambda}{hc}\left(1+\frac{m^2}{2}\right)B = 10\frac{4kTB}{R_L}, \tag{3}$$

so

$$\begin{aligned} P_s &= \frac{hc}{2q^2\eta\lambda B\left(1+\frac{m^2}{2}\right)} \cdot \frac{40kTB}{R_L} \\ &= \frac{(6.63\times10^{-34})(3.0\times10^{8})(40)(1.38\times10^{-23})(300)}{2(1.6\times10^{-19})^2(0.50)(920\times10^{-9}(1.5)(50)} \\ &= 1.867\times10^{-2} = 18.67\ \text{mW}\,. \end{aligned} \tag{4}$$

c. For thermal noise being dominant in a direct detection photodiode, we know that the minimum detectable power is

$$\begin{aligned} P_{\min} &= \frac{hc}{mq\eta\lambda}\sqrt{\frac{2kTB}{R_L}} \\ &= \frac{(6.63\times10^{-34})(3.0\times10^{8})}{(1)(1.6\times10^{-19})(0.50)(920\times10^{-9})}\sqrt{\frac{2(1.38\times10^{-23})(300)(1\times10^{8})}{50}} \\ &= 3.48\times10^{-7} = 0.348\ \text{nW}\,. \end{aligned} \tag{5}$$

17.5. Consider the detector of Problem 4 used in a heterodyne detection application with $f_{IF} = 80$ MHz.

a. Calculate the local oscillator (LO) power that would be required to have the LO-dependent shot noise dominate (by a factor of ten) the dark current-dependent noise.

b. To dominate the thermal noise?

c. Find the minimum detectable power for this detector.

Solution: a. We want the shot noise due to the local oscillator to be ten times the shot noise due to the dark current, i.e., $P_L q\eta\lambda_L/hc = 10\overline{i_d}$, so

$$P_L = \frac{10\overline{i_d}hc}{q\eta\lambda_L} = \frac{(10)(6.63\times10^{-34})(3.0\times10^{8})}{(1.6\times10^{-19})(0.50)(920\times10^{-9})} = 2.70\times10^{-11}\ \text{W} = 27.0\ \text{pW}\,. \tag{6}$$

b. We want the shot noise due to the local oscillator to dominate the thermal noise, so

$$2qB\left(\frac{P_L q\eta\lambda_L}{hc}\right) = 10\cdot\frac{4kTB}{R_L}\,, \tag{7a}$$

and

$$\begin{aligned} P_L &= \frac{40kT}{R_L}\cdot\frac{hc}{2q^2\eta\lambda_L} \\ &= \frac{(40(1.38\times10^{-23})(300)}{500}\cdot\frac{(6.63\times10^{-34})(3.0\times10^{8})}{(2)(1.6\times10^{-19})^2(0.50)(920\times10^{-9})} \\ &= 2.80\times10^{-3}\ \text{W} = 2.80\ \text{mW}\,. \end{aligned} \tag{7b}$$

c. The minimum detectable power for heterodyne detection is

$$P_{\min} = \frac{hcB}{\eta\lambda_L} = \frac{(6.63\times10^{-34})(3.0\times10^{8})(1\times10^{8})}{(0.50)(920\times10^{-9})} = 4.32\times10^{-11}\ \mathrm{W} = 43.2\ \mathrm{pW}\,. \tag{8}$$

18.1. a. Find an expression for the output current of an avalanche photodiode when used in heterodyne detection.

b. Find an expression for the mean-square noise current (neglecting surface dark current and neglecting thermal noise).

c. Find an expression for the amount of local oscillator power P_L that will ensure that the noise is limited by the local-oscillator-dependent shot noise.

Solution: a. We want an expression for the output current of an avalanche photodiode when used with heterodyne detection. Using the results from the photodiode case and adding the gain term, we have

$$i(t) = \frac{P_L q\eta\lambda_L M}{hc} + \frac{2q\eta\lambda_L M\sqrt{P_s P_L}}{hc}\cos(\omega_{\mathrm{IF}}t)\,. \tag{9}$$

b. If we assume $\bar{i}_{\mathrm{d\ surf}} = 0$ and neglect thermal noise, the mean-square noise current will be

$$< i_N^2 >= 2q\bar{i}M^2F(M) = 2q\left(\frac{P_L q\eta\lambda_L}{hc} + \bar{i}_{\mathrm{d\ bulk}}\right)M^{2+x}B\,. \tag{10}$$

c. We want the shot noise due to the local oscillator in the previous result to be ten times the shot noise due to the bulk dark current, so $P_L q\eta\lambda_L/hc = 10 i_{\mathrm{d\ bulk}}$ and

$$P_L = \frac{10 i_{\mathrm{d\ bulk}} hc}{q\eta\lambda_L}\,. \tag{11}$$

Derivation for APD with heterodyne detection

The current out of the APD for the heterodyne case is

$$i(t) = \left(\frac{q\eta\lambda_L}{hc}\right)(M)\left[P_L + 2\sqrt{P_L P_S}\cos\omega_{IF}t\right]\,. \tag{12}$$

(This is found by modifying the result of the photodiode detector used with heterodyne detection.) We find the mean-square signal current as

$$< i_s^2 >= \left(\frac{q^2\eta^2\lambda_L^2}{h^2c^2}\right)M^2(2P_L P_s)\,. \tag{13}$$

The noise is given by

$$< i_N^2 >= 2q\left(\frac{P_L q\eta\lambda_L}{hc}\right)M^{2+x}B + 2q\left(I_{\mathrm{D\ bulk}}\right)M^{2+x}B + 2q\left(I_{\mathrm{D\ surface}}\right)B + \frac{4kTB}{R_L}\,. \tag{14}$$

Now, by increasing P_L, we can always make the first term of the noise dominant,

$$< i_N^2 >= 2q \left(\frac{P_L q \eta \lambda_L}{hc} \right) M^{2+x} B \,, \tag{15}$$

and the signal-to-noise ratio will then be

$$\frac{S}{N} = \frac{P_L \eta \lambda_L}{hcBM^x} \,. \tag{16}$$

To maximize the value of the signal-to-noise ratio, we want to choose $M = 1$ (i.e., no gain at all!). Surprisingly, the APD offers *no advantage* over the pin diode for heterodyne detection when we are limited by the shot noise of the local oscillator.

18.2. Consider an avalanche photodiode operated in a direct detection mode with the following parameters:

Detector	Signal
Bulk dark current: 1 nA	Wavelength: 850 nm
Quantum efficiency: 85%	Modulation index: 0.85
Excess noise factor: $M^{0.5}$	Unmodulated power: -50 dBm
Load resistance: 50 Ω	Noise bandwidth: 1 kHz
$M = 100$	
Surface dark current: 0.01 pA	

Calculate the signal-to-noise ratio in dB:

a. . . . if the signal-dependent shot noise is dominant,

b. . . . if the shot noise due to the surface dark current is dominant,

c. . . . if the shot noise due to the bulk dark current is dominant,

d. . . . if the thermal noise is dominant, and

e. . . . if all noise sources are present.

Solution: a. We want the signal-to-noise ratio if the shot noise due to the signal is assumed dominant. The power is

$$P_s = -50 \text{ dBm} \Rightarrow 10^{-5} \text{ mW} = 1 \times 10^{-8} \text{ W} \,. \tag{17}$$

The mean-square signal current is

$$\begin{aligned} < i_s^2 > &= \frac{2m^2 M^2 \left(P_s q \eta \lambda\right)^2}{(hc)^2} \qquad (18a) \\ &= \frac{2(0.85)^2 (100)^2 \left((1 \times 10^{-8})(1.6 \times 10^{-19})(0.85)(850 \times 10^{-9})\right)^2}{((6.63 \times 10^{-34})(3.0 \times 10^8))^2} \\ &= 4.88 \times 10^{-13} \text{ A}^2 \,. \end{aligned}$$

The mean-square noise current is

$$
\begin{aligned}
< i_N^2 > &= 2q\left[\frac{P_s q\eta\lambda}{hc}\left(1+\frac{m^2}{2}\right)\right] BM^{2+x} \qquad (18b)\\
&= 2(1.6\times 10^{-19})\left[\frac{(1\times 10^{-8})(1.6\times 10^{-19})(0.85)(850\times 10^{-9})\left(1+\frac{(0.85)^2}{2}\right)}{(6.63\times 10^{-34})(3.0\times 10^{8})}\right]\\
&\quad \times (1\times 10^3)(100)^{2.5}\\
&= 2.53\times 10^{-19}\ \mathrm{A}^2\,.
\end{aligned}
$$

Hence, the SNR is

$$\frac{\mathrm{S}}{\mathrm{N}} = \frac{< i_s^2 >}{< i_N^2 >} = \frac{4.82\times 10^{-13}}{2.52\times 10^{-19}} = 1.928\times 10^6 \Rightarrow 62.9\ \mathrm{dB}\,. \qquad (18c)$$

b. If the shot noise due to the surface dark current is dominant, the mean-square noise current is

$$< i_N^2 >= 2q\overline{i_{\mathrm{d\ surf}}}B = 2(1.6\times 10^{-19})(1\times 10^{-14})(1\times 10^3) = 3.2\times 10^{-30}\ \mathrm{A}^2\,. \qquad (19a)$$

The SNR, then, is

$$\frac{\mathrm{S}}{\mathrm{N}} = \frac{< i_s^2 >}{< i_N^2 >} = \frac{4.82\times 10^{-13}}{3.2\times 10^{-30}} = 1.506\times 10^{17} \Rightarrow 171.8\ \mathrm{dB}\,. \qquad (19b)$$

c. If the shot noise due to the bulk dark current is dominant, the mean-square noise current is

$$< i_N^2 >= 2q\overline{i_{\mathrm{d\ bulk}}}BM^{2.5} = 2(1.6\times 10^{-19})(1\times 10^{-9})(1\times 10^3)(100)^{2.5} = 3.2\times 10^{-20}\ \mathrm{A}^2\,. \qquad (20a)$$

The SNR, then, is

$$\frac{\mathrm{S}}{\mathrm{N}} = \frac{< i_s^2 >}{< i_N^2 >} = \frac{4.82\times 10^{-13}}{3.2\times 10^{-20}} = 1.506\times 10^{7} \Rightarrow 71.8\ \mathrm{dB}\,. \qquad (20b)$$

d. If the thermal noise is assumed dominant, then the mean-square noise current is

$$< i_N^2 >= \frac{4kTB}{R_L} = \frac{4(1.38\times 10^{-23})(300)(1\times 10^3)}{50} = 3.3\times 10^{-19}\ \mathrm{A}^2 \qquad (21a)$$

and the SNR is

$$\frac{\mathrm{S}}{\mathrm{N}} = \frac{< i_s^2 >}{< i_N^2 >} = \frac{4.82\times 10^{-13}}{3.3\times 10^{-19}} = 1.461\times 10^{6} \Rightarrow 61.7\ \mathrm{dB}\,. \qquad (21b)$$

e. If all of the noise sources are present,

$$
\begin{aligned}
< i_N^2 >_{\mathrm{total}} &=< i_N^2 >_a + < i_N^2 >_b + < i_N^2 >_c + < i_N^2 >_d \qquad (22a)\\
&= 2.52\times 10^{-19} + 3.2\times 10^{-30} + 3.2\times 10^{-20} + 3.3\times 10^{-19}\\
&= 6.14\times 10^{-19}\ \mathrm{A}^2\,.
\end{aligned}
$$

and

$$\frac{\mathrm{S}}{\mathrm{N}} = \frac{< i_s^2 >}{< i_N^2 >} = \frac{4.82 \times 10^{-13}}{6.14 \times 10^{-19}} = 7.85 \times 10^5 \Rightarrow 59.0 \text{ dB}. \tag{22b}$$

18.3. Consider the detector of the previous problem.

a. Calculate the optimum value of M.

b. How many dB increase in the S/N can be realized by operating the detector at M_{opt} compared with operating it at $M = 100$?

<u>Solution:</u> a. The optimum value of M is

$$M_{\text{opt}}^{2+x} = \frac{2q\overline{i_{\text{d surf}}} + \dfrac{4kT}{R_L}}{xq\left(\overline{i_s}\big|_{M=1} + \overline{i}_{\text{d bulk}}\right)} \tag{23a}$$

where

$$\begin{aligned}\overline{i_s}\big|_{M=1} &= \frac{P_s q\eta\lambda}{hc}\left(1 + \frac{m^2}{2}\right) \\ &= \frac{(1 \times 10^{-8})(1.6 \times 10^{-19})(0.85)(850 \times 10^{-9})}{(6.63 \times 10^{-34})(3.0 \times 10^8)}\left(1 + \frac{(0.85)^2}{2}\right) \\ &= 7.89 \times 10^{-9} \text{ A}\end{aligned} \tag{23b}$$

and

$$M_{\text{opt}}^{2.5} = \frac{2(1.6 \times 10^{-19})(1 \times 10^{-14}) + \dfrac{4(1.38 \times 10^{-23})(300)}{50}}{(0.5)(1.6 \times 10^{-19})\left(7.89 \times 10^{-9} + 1 \times 10^{-9}\right)} = 4.65 \times 10^5 \tag{23c}$$

and, hence,

$$M_{\text{opt}} = (4.65 \times 10^3)^{(1/2.5)} = (4.65 \times 10^5)^{0.4} = 185.0. \tag{23d}$$

b. At the optimum gain of $M = M_{\text{opt}} = 185.0$, the mean-square noise current is

$$\begin{aligned}< i_N^2 > &= 2q\,\overline{i_s}\big|_{M=1} BM^{2.5} + 2q\overline{i_{\text{d bulk}}}BM^{2.5} + 2q\overline{i_{\text{d surf}}}B + \frac{4kTB}{R_L} \\ &= 2(1.6 \times 10^{-19})(7.89 \times 10^{-9})(1 \times 10^3)(185.0)^{2.5} \\ &\quad + 2(1.6 \times 10^{-19})(1 \times 10^{-9})(1 \times 10^3)(185.0)^{2.5} \\ &\quad + 2(1.6 \times 10^{-19})(1 \times 10^{-9})(1 \times 10^3) \\ &\quad + \frac{4(1.38 \times 10^{-23})(300)(1 \times 10^3)}{50} \\ &= 1.17 \times 10^{-18} + 1.492 \times 10^{-19} + 3.204 \times 10^{-25} + 3.31 \times 10^{-22} \\ &= 1.267 \times 10^{-18} \text{ A}^2,\end{aligned} \tag{24a}$$

the mean-square signal current is

$$< i_s^2 > = 2m^2 \left(\frac{P_s q \eta \lambda}{hc} \right)^2 M_{\text{opt}}^2 \tag{24b}$$
$$= 2(0.85)^2 \left(\frac{(10 \times 10^{-9})(1.6 \times 10^{-19})(0.85)(850 \times 10^{-9})}{(6.63 \times 10^{-34})(3.0 \times 10^8)} \right)^2 (185.0)^2$$
$$= 1.679 \times 10^{-12} \text{ A}^2 ,$$

and the signal-to-noise ratio is

$$\frac{\text{S}}{\text{N}} = \frac{< i_s^2 >}{< i_N^2 >} = \frac{1.679 \times 10^{-12}}{1.267 \times 10^{-18}} = 1.053 \times 10^6 \Rightarrow 60.2 \text{ dB} . \tag{24c}$$

At M_{opt} the S/N is 60.2 dB compared with the S/N of 59.0 when operated with $M = 100$. The improvement of 1.2 dB is minor, but present.

19.1. Consider a turbojet engine with an exhaust temperature of 485 C and an area of 3700 cm^2 [Ref. 19.1]. Assume an emissivity of 0.9. Calculate

a. the radiance,

b. the radiant intensity for all wavelengths, and

c. the radiant intensity that lies between 3.2 μm and 4.8 μm.

Solution: a. The radiance N is

$$N = \frac{W}{\pi} = \frac{\epsilon \sigma T^4}{\pi} = \frac{(0.9)(5.67 \times 10^{-12})(758)^4}{\pi} = 5.36 \times 10^{-1} \text{ W} \cdot \text{cm}^{-2} \cdot \text{sr}^{-1} . \tag{25}$$

b. The radiant intensity is

$$J = NA = (5.36 \times 10^{-1})(3700) = 1.984 \times 10^3 \text{ W} \cdot \text{sr}^{-1} . \tag{26}$$

c. From our figure in the notes (or from a blackbody slide-rule), we find that ...

i. ... $\approx 15.5\%$ of the power lies below 3.2 μm.

ii. ... $\approx 42.5\%$ of the power lies below 4.8 μm.

So 27% ($= 42.5 - 15.5\%$) of the power lies between 3.2 μm and 4.8 μm, and the radiant intensity in that wavelength region is

$$J = (0.27)(J_{\text{total}}) = (0.27)(1.984 \times 10^3) = 536 \text{ W} \cdot \text{sr}^{-1} . \tag{27}$$

19.2. Repeat the calculations of Problem 1 for a fanjet engine operating at 445 C with a 3500 cm^2 area [Ref. 19.1].

Solution: a. The radiance N is

$$N = \frac{W}{\pi} = \frac{\epsilon \sigma T^4}{\pi} = \frac{(0.9)(5.67 \times 10^{-12})(718)^4}{\pi} = 4.31 \times 10^{-1} \text{ W} \cdot \text{cm}^{-2} \cdot \text{sr}^{-1} . \tag{28}$$

b. The radiant intensity J is

$$J = NA = (4.31 \times 10^{-1})(3500) = 1.508 \times 10^3 \text{ W} \cdot \text{sr}^{-1} . \tag{29}$$

(c.) From our figure in the notes (or from a blackbody slide-rule), we find that ...

i. ... $\approx 12.8\%$ of the power lies below 3.2 μm.

ii. ... $\approx 37.5\%$ of the power lies below 4.8 μm.

So 24.7% (= 37.5−12.8%) of the power lies between 3.2 μm and 4.8 μm, and the radiant intensity in that wavelength region is

$$J = (0.247)(J_{\text{total}}) = (0.247)(1.508 \times 10^3) = 373 \text{ W} \cdot \text{sr}^{-1} . \tag{30}$$