EC 4210 Solutions

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Assignment 3

15.3. Calculate the minimum secondary emission gain δ required for an 8-stage photomultiplier to provide an overall current gain G of 10^5.

Solution: The stage gain δ required to produce a device total gain of 1×10^5 over eight stages of gain is

$$\delta^N = G \tag{1a}$$

$$\delta^8 = 1 \times 10^5 \tag{1b}$$

$$\begin{aligned} \delta &= (1 \times 10^5)^{(1/8)} = 4.21 \text{ (but } \delta \text{ must be an integer, so)} \\ &= 5\,. \end{aligned} \tag{1c}$$

15.4. Equation 15.26 on page 239 is an expression for the signal-to-noise ratio when the shot noise is dominant. Find an expression for the gain G required to make the shot noise dominate the thermal noise.

Solution: The total mean-square-noise current is given by

$$< i_N^2 >= 2qG^2(\bar{i}_c + i_d)B + \frac{4KTB}{R}\,. \tag{2a}$$

We want the first term to dominate the second, so

$$2qG^2(\bar{i}_c + i_d)B \gg \frac{4KTB}{R} \tag{2b}$$

and

$$G^2 \gg \frac{4KTB}{2q(\bar{i}_c + i_d)BR} \tag{2c}$$

or

$$G \gg \sqrt{\frac{4KTB}{2q(\bar{i}_c + i_d)BR}} = \sqrt{\frac{4KTB}{2q(\mathcal{R}_0 P + i_d)BR}}\,. \tag{2d}$$

15.5. Consider a photomultiplier with the following characteristics:

Quantum efficiency = 10%
Wavelength: 800 nm
Gain: 10^5
$i_d = 10^{-12}$ A
Power (in unmodulated signal): 1 nW
$m = 1$
$R_L = 1K\Omega$
$B = 1$ Hz
$T_{\text{noise}} = 400K$

a. Calculate the values of $\overline{i}_c$ and $\overline{i}_{out}$.

b. Calculate the value of G that would be required to ensure that the mean square noise current due to the shot noise is ten times the thermal noise. Does the device have enough gain to ensure this mode of operation?

c. For $G = 10^5$, calculate the mean square noise current due to the shot noise associated with the signal.

d. For $G = 10^5$, calculate the mean square noise current of the shot noise associated with the dark current.

e. For $G = 10^5$, calculate the signal-to-noise ratio in dB.

Solution: We have a PMT with $\eta = 0.10$, $\lambda = 800 \times 10^{-9}$ m, $G = 1 \times 10^5$, $i_d = 1 \times 10^{-12}$ A, $P = 1 \times 10^{-9}$ W, $m = 1$, $R_L = 1 \times 10^3$ ohms, $B = 1$ Hz, and $T_{\text{Noise}} = 400$K.

a. The dc photocathode current $\overline{i_c}$ is

$$\overline{i_c} = \frac{Pq\eta\lambda}{hc}\left(1 + \frac{m^2}{2}\right) = \frac{(1 \times 10^{-9})(1.6 \times 10^{-19})(0.10)(800 \times 10^{-9})}{(6.63 \times 10^{-34})(3.0 \times 10^8)}\left(\frac{3}{2}\right) = 9.65 \times 10^{-11}\ \text{A}\,. \tag{3a}$$

The output current is

$$\overline{i_{\text{out}}} = G\overline{i_c} = (1 \times 10^5)(9.65 \times 10^{-11}) = 9.65 \times 10^{-6}\ \text{A}\,. \tag{3b}$$

b. We want

$$2qG^2(\overline{i_c} + i_d)B = 10\frac{4kTB}{R_L} \tag{4a}$$

$$G \geq \sqrt{\frac{40kTB}{R_L}\frac{1}{2q(\overline{i_c} + i_d)}B} \tag{4b}$$

$$\geq \sqrt{\frac{(40)(1.38 \times 10^{-23})(400)(1)}{(1 \times 10^3)(2)(1.6 \times 10^{-19})(9.65 \times 10^{-11} + 1 \times 10^{-12})(1)}}$$

$$\geq 2.66 \times 10^3\,.$$

We conclude that our gain of 1×10^5 *is* large enough to make the shot noise dominant over the thermal noise.

c. The mean-square shot noise current for the signal current with $G = 1 \times 10^5$ is

$$< i_{N1}^2 >= 2q\overline{i_c}BG^2 = 2(1.6 \times 10^{-19})(9.65 \times 10^{-11})(1)(1 \times 10^5)^2 = 3.08 \times 10^{-19}\ \text{A}^2\,. \tag{5}$$

d. The mean-square shot noise current for the dark current with $G = 1 \times 10^5$ is

$$< i_{N2}^2 >= 2q\overline{i_d}BG^2 = 2(1.6 \times 10^{-19})(1 \times 10^{-12})(1)(1 \times 10^5)^2 = 3.20 \times 10^{-21}\ \text{A}^2\,. \tag{6}$$

e. The signal-to-noise ratio, S/N, is

$$\frac{\mathrm{S}}{\mathrm{N}} = \frac{< i_s^2 >}{< i_{N1}^2 > + < i_{N2}^2 >} \tag{7a}$$

$$< i_s^2 > = 2m^2 \left[\frac{GP\lambda\eta q}{hc} \right]^2 \tag{7b}$$

$$= 2(1)^2 \left[\frac{(1 \times 10^5)(1 \times 10^{-9})(800 \times 10^{-9})(0.10)(1.6 \times 10^{-19})}{(6.63 \times 10^{-34})(3.0 \times 10^8)} \right]^2$$

$$= 8.26 \times 10^{-11} \ \mathrm{A}^2 \, .$$

$$\frac{\mathrm{S}}{\mathrm{N}} = \frac{8.26 \times 10^{-11}}{3.08 \times 10^{-19} + 3.2 \times 10^{-21}} = 2.65 \times 10^8 \Rightarrow 84.2 \ \mathrm{dB} \, .$$

This is an enormous signal-to-noise ratio.