EC 4210 Solutions

Assignment 2

13.4. Consider a Bragg cell of lead molybdate with dimensions of 5 cm wide x 2 cm high by 8 cm long. A transducer is bonded onto the 5 cm x 2 cm face. The transducer center frequency is 80 MHz with a fractional bandwidth of 50%. The cell will work with an argon laser operating at a wavelength of 488 nm.

a. Calculate the Bragg angle at the center frequency, the upper frequency of the transducer, and the lower frequency of the transducer.

b. If the laser beam is 1 mm wide in the interaction region, calculate the acoustic transit time.

c. Calculate the maximum modulation bandwidth associated with the acoustic transit time.

d. Calculate the minimum laser beamwidth required to use the full bandwidth of the transducer.

e. Calculate the minimum) acoustic power that would be required to deflect 50% of the incident laser beam power.

f. Calculate the range of the deflection angle $\Delta\theta$ for the full bandwidth of the transducer, if this Bragg cell is used as a beam deflector.

Solution: The center frequency is 80 MHz. Since the bandwidth of the transducer is 50% of the center frequency (as a rule of thumb), the bandwidth Δf is

$$\Delta f = 50\% \times f_c = 0.5(80 \times 10^6) = 40 \times 10^6 \text{ Hz} \Rightarrow 40 \text{ MHz}. \tag{1}$$

The upper frequency is $f_c + (\Delta f/2) = 100$ MHz and the lower frequency is $f_c - (\Delta f/2) = 60$ MHz.

a. The Bragg angle at the f_c is

$$\theta_B = \frac{\lambda f_c}{2v_s} = \frac{(488 \times 10^{-9})(80 \times 10^6)}{2(3.75 \times 10^3)} = 5.2 \times 10^{-3} \text{ radians} \Rightarrow 0.298^\circ. \tag{2a}$$

At the upper frequency

$$\theta_B = \frac{\lambda f_u}{2v_s} = \frac{(488 \times 10^{-9})(100 \times 10^6)}{2(3.75 \times 10^3)} = 6.51 \times 10^{-3} \text{ radians} \Rightarrow 0.373^\circ. \tag{2b}$$

At the lower frequency

$$\theta_B = \frac{\lambda f_l}{2v_s} = \frac{(488 \times 10^{-9})(60 \times 10^6)}{2(3.75 \times 10^3)} = 3.904 \times 10^{-3} \text{ radians} \Rightarrow 0.224^\circ. \tag{2c}$$

b. If $d = 1 \times 10^{-3}$ m, we find the transit time τ as

$$\tau = \frac{d}{v_s} = \frac{1 \times 10^{-3}}{3.75 \times 10^3} = 2.67 \times 10^{-5}\ \mathrm{s} \Rightarrow 26.7\ \mu\mathrm{s}\,. \tag{3}$$

c. The maximum frequency limit set by the transit time is

$$f_{\max} = \frac{1}{\tau} = \frac{1}{0.267 \times 10^{-6}} = 3.75\ \mathrm{MHz}\,. \tag{4}$$

d. To keep up with the desired 100 MHz upper frequency of our system requires

$$f_{\max} = \frac{1}{\tau} = \frac{1}{\dfrac{d}{v_s}} \tag{5a}$$

so

$$d = \frac{v_s}{f_{\max}} = \frac{3.75 \times 10^3}{100 \times 10^6} = 37.5 \times 10^{-6}\ \mathrm{m} = 37.5\ \mu\mathrm{m}\,. \tag{5b}$$

e. The acoustic power P_{ac} for 50% deflection efficiency is

$$\frac{P_d}{P_0} = \sin^2\left[\frac{\pi L\sqrt{M_2 I_{\mathrm{ac}}}}{\sqrt{2}\lambda}\right] \tag{6a}$$

so

$$M_2 I_{\mathrm{ac}} = \left[\sin^{-1}\left(\sqrt{\frac{P_d}{P_0}}\right)^2\right]^2 \left[\frac{\sqrt{2}\lambda}{\pi L}\right]^2 \tag{6b}$$

and

$$\begin{aligned} P_{\mathrm{ac}} = I_{\mathrm{ac}} A &= \frac{A}{M_2}\left(\frac{\sqrt{2}\lambda}{\pi L}\right)^2 \left(\sin^{-1}\sqrt{\frac{P_d}{P_0}}\right)^2 \\ &= \frac{0.001}{(0.22)(1.576 \times 10^{-13})}\left(\frac{\sqrt{2}(488 \times 10^{-9})}{\pi(5 \times 10^{-2})}\right)^2 \left(\sin^{-1}\sqrt{0.50}\right)^2 = 0.134\ \mathrm{W}\,. \end{aligned} \tag{7}$$

(Here, A is the cross-section area of the acoustic beam.)
f. The deflection angle of the cell is

$$\Delta\theta = \frac{\lambda \Delta f}{v_s} = \frac{(488 \times 10^{-9})(40 \times 10^6)}{3.75 \times 10^3} = 5.21 \times 10^{-3}\ \text{radians} \Rightarrow 0.298^\circ\,. \tag{8}$$

13.5. A tellurium dioxide acousto-optic beam deflector uses a transducer with a center frequency of 100 MHz. If we want to design a 2,000 spot deflector, find the diameter of the laser beam at the position of the light-sound interaction.

Solution: The bandwidth of the transducer can be estimated as

$$\mathrm{BW} = \frac{f_c}{2} = \frac{100 \times 10^6}{2} = 50 \text{ MHz}\,. \tag{9}$$

We want $N = 2000$, so

$$N = \mathrm{BW}\,\tau = \mathrm{BW}\left(\frac{d}{v_s}\right)\,, \tag{10a}$$

hence,

$$d = \frac{N v_s}{\mathrm{BW}} = \frac{(2000)(0.63 \times 10^3)}{50 \times 10^6} = 2.52 \times 10^{-2} \text{ m} \Rightarrow 2.52 \text{ cm}\,. \tag{10b}$$

15.1. Find the required work function (in joules and eV) for a photomultiplier to have a long wavelength cutoff of (a.) 1 μm and (b.) 10 μm.

Solution: The work function of a material is related to the maximum wavelength by $\phi = \frac{hc}{\lambda_{\max}}$.
a. For $\lambda_{\max} = 1\ \mu$m, we need

$$\phi = \frac{hc}{\lambda_{\max}} = \frac{(6.63 \times 10^{-34})(3.0 \times 10^8)}{1 \times 10^{-6}} = 1.992 \times 10^{-19} \text{ joules} \Rightarrow 1.245 \text{ eV}\,. \tag{11}$$

b. For $\lambda_{\max} = 10\ \mu$m, we need a work function that is one-tenth of the value in part a, so $\phi = 0.124$ eV.

15.2. Consider a photomultiplier operating with 3 mW of input power at 920 nm that produces 200 μA of cathode current.

a. Calculate the number of photons per second in the light.

b. Calculate the number of electrons per second that are freed at the photocathode.

Solution: a. The number of photons per second N' in the beam is

$$N' = \frac{P}{\frac{hc}{\lambda}} = \frac{3 \times 10^{-3}}{\frac{(6.63 \times 10^{-34})(3.0 \times 10^8)}{920 \times 10^{-9}}} = 1.389 \times 10^{16} \text{ photons} \cdot \text{s}^{-1}\,. \tag{12}$$

b. The number of electrons per second M' freed at the photocathode is

$$M' = \frac{I}{q} = \frac{200 \times 10^{-6}}{1.6 \times 10^{-19}} = 1.25 \times 10^{15} \text{ electrons} \cdot \text{s}^{-1}\,. \tag{13}$$

The quantum efficiency η is

$$\eta = \frac{M'}{N'} = \frac{1.389 \times 10^{16}}{1.25 \times 10^{15}} = 0.0898 \Rightarrow 8.98\%\,. \tag{14}$$

The responsivity without gain, $\mathcal{R}_0$, is

$$\mathcal{R}_0 = \frac{I}{P} = \frac{200 \times 10^{-6}}{3 \times 10^{-3}} = 6.67 \times 10^{-2} = 66.6 \text{ mA} \cdot \text{W}^{-1}\,. \tag{15}$$