EC 4210 Solutions

Assignment 1

13.1. Consider a KD*P crystal immersed in an electric field of $1.0 \times 10^{10^5}$ V/m along the x' axis. The crystal is to be used with a 500 nm source. (See 13.1 on page 199 for the crystal properties. Note that $n_f = n_s = n$ for this crystal.)

a. Calculate $\Delta n_{y'} = n_{y'} - n$.

b. Calculate θ if the crystal length D is 5 cm.

c. Calculate V_π for this crystal.

This crystal is to be used in a electro-optical phase modulator.

d. When $V = 0$ volts, calculate the phase change of a plane wave polarized along the y' axis that propagates through the crystal ($D = 5$ cm).

e. Calculate the voltage required to cause an additional $+10°$ phase shift of this plane wave.

Solution:

a. We find

$$\Delta n_{y'} = n_{y'} - n = -\frac{n_s^3 r_{63} E_{x'}}{2} = -\frac{(1.5)^3(23.6 \times 10^{-12})(1 \times 10^5)}{2} = -3.98 \times 10^{-6}\,. \tag{1a}$$

We note that this is a very small fractional change in the index of refraction, but it still can have major effects on the light wave.

Similarly,

$$\Delta n_{z'} = +|\Delta n_{y'}| = +3.98 \times 10^{-6}\,. \tag{1b}$$

b. For $D = 5 \times 10^{-2}$ m and $\lambda = 500 \times 10^{-9}$ m, we have

$$\begin{aligned}\theta &= \frac{2\pi\nu n_s^3 r_{63} E_{x'} D}{c} = \frac{2\pi n_s^3 r_{63} E_{x'} D}{\lambda} = \frac{2\pi(1.5)^3(23.6 \times 10^{-12})(1 \times 10^5)(5 \times 10^{-2})}{500 \times 10^{-9}} \\ &= 5.00 \text{ radians} \Rightarrow 287°\,.\end{aligned} \tag{2}$$

c. The value of V_π is

$$V_\pi = \frac{\lambda}{2 n_s^3 r_{63}} = \frac{500 \times 10^{-9}}{(2)(1.5)^3(23.6 \times 10^{-12})} = 3139 \text{ volts}\,. \tag{3}$$

d. The phase shift angle is

$$\phi_{y'} = -\frac{\omega n_s D}{c} + \frac{\pi V}{2V_\pi} \tag{4a}$$

$$\begin{aligned}\phi_{y'}|_{V=0} &= -\frac{\omega n_s D}{c} = -\frac{2\pi n_s D}{\lambda} = -\frac{2\pi(1.5)(5\times10^{-2})}{500\times10^{-9}} \\ &= -9.42\times10^{5} \text{ radians} \Rightarrow 5.4\times10^{7\,\circ}\,.\end{aligned} \tag{4b}$$

e. The voltage for a 10° phase shift is

$$\frac{\pi V(10^\circ)}{2V_\pi} = 10^\circ = 1.745\times10^{-1} \text{ radians} \tag{5a}$$

or

$$V(10^\circ) = \frac{(1.745\times10^{-1})(2)V_\pi}{\pi} = \frac{(1.745\times10^{-1})(2)(3139}{\pi} = 348.8 \text{ volts}\,. \tag{5b}$$

13.2. Suppose that a KD*P crystal that is 5 mm x 5 mm x 5 cm is to be used as a transverse electro-optic modulator at 500 nm.

a. Calculate the half-wave voltage.

b. Find an expression for the ratio of the half-wave voltage of a longitudinal modulator to the half-wave voltage of a transverse modulator with the same length D.

Solution: a. The half-wave voltage V_π is

$$V_\pi = \frac{\lambda}{n_s^3 r_{63}}\frac{d}{D} = \left(\frac{500\times10^{-9}}{(1.5)^3(23.6\times10^{-12})}\right)\left(\frac{5\times10^{-3}}{5\times10^{-2}}\right) = 627 \text{ volts}\,. \tag{6}$$

b. The ratio of the voltages is

$$\frac{V_{\pi\ \text{long}}}{V_{\pi\ \text{trans}}} = \frac{\dfrac{\lambda}{2n_s^3 r_{63}}}{\left(\dfrac{\lambda}{n_s^3 r_{63}}\right)\left(\dfrac{d}{D}\right)} = \frac{D}{2d}\,. \tag{7}$$

13.3. Consider the longitudinal electro-optic irradiance modulator shown in Fig. 13.18 on page 218 (repeated here as Fig. 1. (The z' axis is horizontal; the y' axis is vertical.) The polarization axis of the input polarizer makes an angle ϕ with respect to the vertical axis. The polarization axis of the output linear polarizer is perpendicular to that of the input polarizer.

Derive an expression for I_{out}/I_{in} in terms of ϕ and the retardation angle θ for this geometry. (Be sure to clearly show all steps required in the derivation.)

Solution: We begin by decomposing the input wave at the face of the crystal into two waves, one aligned along the y' axis and one aligned along the z' axis. We will represent these waves by phasors.

$$\tilde{E}_{y'} = E_0 \cos\phi\, e^{j0} \tag{8a}$$

$$\tilde{E}_{z'} = E_0 \sin\phi\, e^{j0} \tag{8b}$$

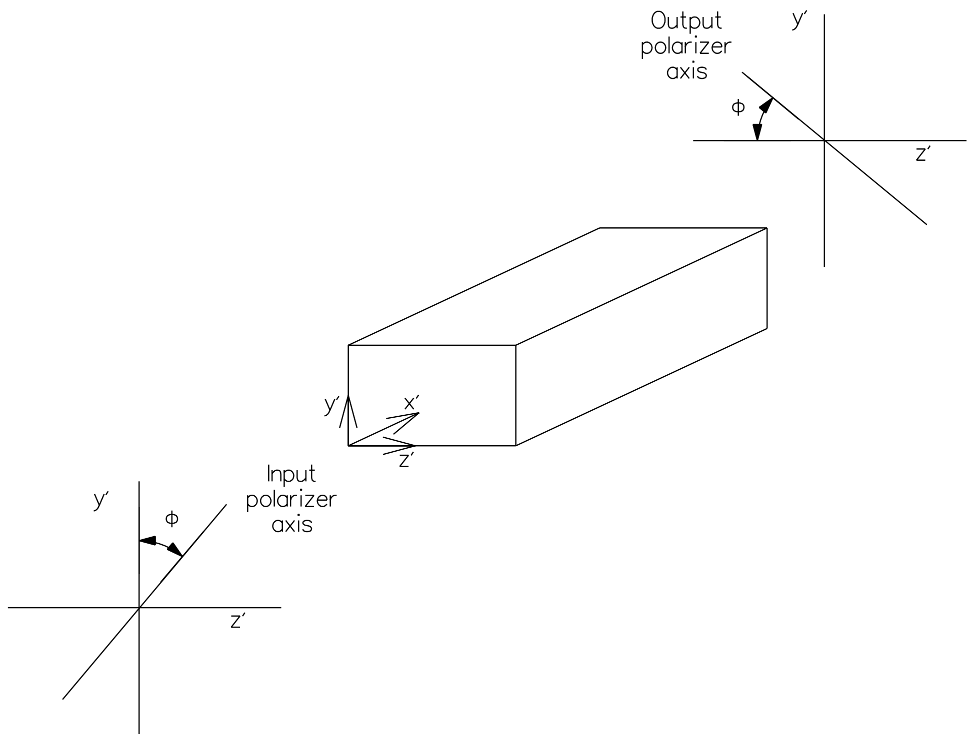

Figure 1: Geometry for Prob. 13.3. Crystal axes geometry, input polarizer orientation at an angle ϕ from the vertical, and output polarizer orientation (orthogonal to input polarizer).

At the output of the crystal, we can write the phasors as

$$\tilde{E}_{y'} = E_0 \cos\phi \, e^{-j\frac{\omega n_s D}{c}} e^{+j\frac{\omega n_s^3 r_{63} V}{2c}} \tag{9a}$$

$$\tilde{E}_{z'} = E_0 \sin\phi \, e^{-j\frac{\omega n_s D}{c}} e^{-j\frac{\omega n_s^3 r_{63} V}{2c}} \tag{9b}$$

The phase difference between these waves at the output face is still

$$\Delta\phi = \phi_{y'} - \phi_{z'} = \frac{\omega n_s^3 r_{63} V}{c} = \theta \, . \tag{10}$$

We now have to find the components that lie along the axis of the output polarizer (see Fig. 2).

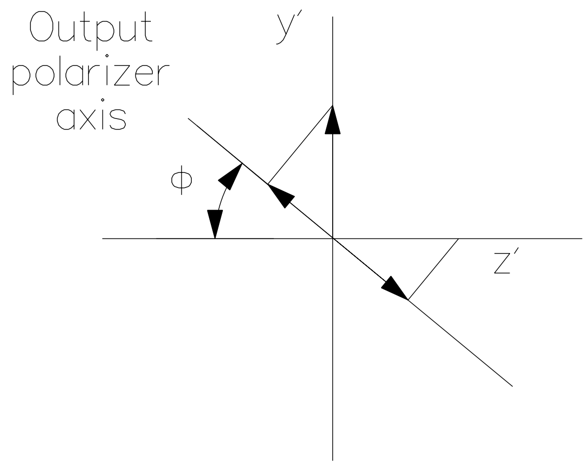

Figure 2: Problem 13.3. Geometry required to calculate the values of components aligned along the output polarizer axis.

$$\begin{aligned}
\tilde{E}_{\text{pol}} &= -\tilde{E}_{y'}\sin\phi + \tilde{E}_{z'}\cos\phi \\
&= -E_0\cos\phi\sin\phi\, e^{-j\frac{\omega n_s D}{c}} e^{j\frac{\theta}{2}} + E_0\sin\phi\cos\phi e^{-j\frac{\omega n_s D}{c}} e^{-j\frac{\theta}{2}} \\
&= E_0\sin\phi\cos\phi\, e^{-j\frac{\omega n_s D}{c}}\left(e^{-j\frac{\theta}{2}} - e^{+j\frac{\theta}{2}}\right) \\
&= -E_0\sin\phi\cos\phi\, e^{-j\frac{\omega n_s D}{c}}\left(e^{+j\frac{\theta}{2}} - e^{-j\frac{\theta}{2}}\right) \\
&= -2jE_0\sin\phi\cos\phi\, e^{-j\frac{\omega n_s D}{c}}\left(\frac{e^{+j\frac{\theta}{2}} - e^{-j\frac{\theta}{2}}}{2j}\right) \\
&= -2jE_0\sin\phi\cos\phi\, e^{-j\frac{\omega n_s D}{c}}\sin\left(\frac{\theta}{2}\right)\,.
\end{aligned} \tag{11}$$

Finding the magnitude squared of the field, we have

$$|\tilde{E}_{\text{pol}}|^2 = 4E_0^2\sin^2\phi\cos^2\phi\sin^2\left(\frac{\theta}{2}\right) = E_0^2\sin^2(2\phi)\sin^2\left(\frac{\theta}{2}\right) \tag{12}$$

The ratio of the output irradiance of the modulator to the input irradiance is

$$\frac{I_{\text{out}}}{I_{\text{in}}} = \sin^2(2\phi)\sin^2\left(\frac{\theta}{2}\right)\,. \tag{13}$$