Chapter 8

Assignment 8 Solutions

5.13 A step–index fiber is excited by a laser with an emission pattern approximated by $\cos^7\theta$. Calculate the improvement factor relative to a Lambertian source.

Solution: The improvement in coupling efficiency for an emitter with a $\cos^m\theta$ dependence over a $\cos\theta$ dependence is a factor of $(m+1)/2$. For $m = 7$, the improvement would be a factor of $(7+1)/2 = 4 \Rightarrow 6.0$ dB.

5.14. Consider a graded–index fiber ($a = 30$ μm, $\Delta = 1.5\%$, $g = 1.95$, and $n_1 = 1.45$) excited by a surface emitting LED ($B_0 = 2.0 \times 10^2$ W·cm^{-2}·sr^{-1} and radius of 50 μm). Calculate...

a. ...the power emitted by the source,

b. ...the power coupled into the fiber, and

c. ...the coupling losses (in dB).

Solution: We will need the value of NA(0) found from

$$\mathrm{NA}(0) = n_1\sqrt{2\Delta} = (1.45)\sqrt{(2)(0.015)} = 0.251\,. \tag{8.1}$$

a. The power P_s from the source is

$$P_s = \pi^2 r_s^2 B_0 = \pi^2(50\times10^{-4})^2(200) = 4.93\times10^{-2}\ \mathrm{W} = 49.3\ \mathrm{mW}\,. \tag{8.2}$$

(Note that I have calculated the source area in units of cm^2; B_0 was expressed in terms of those area units.)

b. We need to find the coupling efficiency in the next part first...

c. Since $r_s > a$, the coupling efficiency η is

$$\begin{aligned}\eta &= (\mathrm{NA}(0))^2\left[\frac{g}{g+2}\left(\frac{a}{r_s}\right)^2\right] = (0.251)^2\left[\frac{1.95}{1.95+2}\left(\frac{30}{50}\right)^2\right]\\ &= 0.01120 \Rightarrow 1.12\% \Rightarrow 19.5\ \mathrm{dB}\,.\end{aligned} \tag{8.3}$$

The answer to Part b, then, is

$$P_f = \eta P_s = (0.01120)(4.93 \times 10^{-2}) = 5.52 \times 10^{-4} \text{ W} = 552 \ \mu\text{W}\,. \tag{8.4}$$

5.16. a. Calculate the loss due to reflections in an interface that has the light go from GaAs ($n = 3.5$), through air ($n = 1.0$), and into a fiber ($n = 1.5$).

b. Calculate the transmission if the air is replaced with a medium (such as epoxy) that matches the index of the glass.

Solution: a. The calculate the combined transmission as the product of the individual transmissions.

$$\begin{aligned} T &= \left(1 - \left[\frac{n_1 - n_2}{n_1 + n_2}\right]^2\right)\left(1 - \left[\frac{n_2 - n_3}{n_2 + n_3}\right]^2\right) \\ &= \left(1 - \left[\frac{3.5 - 1}{3.5 + 1}\right]^2\right)\left(1 - \left[\frac{1 - 1.5}{1 + 1.5}\right]^2\right) \\ &= 0.664 \Rightarrow 66.4\%\,. \end{aligned} \tag{8.5}$$

b. Replacing n_2 by 1.5 instead of 1,

$$\begin{aligned} T &= \left(1 - \left[\frac{n_1 - n_2}{n_1 + n_2}\right]^2\right)\left(1 - \left[\frac{n_2 - n_3}{n_2 + n_3}\right]^2\right) \\ &= \left(1 - \left[\frac{3.5 - 1.5}{3.5 + 1.5}\right]^2\right)\left(1 - \left[\frac{1.5 - 1.5}{1.5 + 1.5}\right]^2\right) \\ &= 0.840 \Rightarrow 84.0\%\,. \end{aligned} \tag{8.6}$$

We have improved the coupling efficiency. Can find the coupling efficiency if n_2 is matched to the source index instead of the fiber? Is there an optimum matching index? (It can be shown that the optimum value of n_2 is $n_2 = \sqrt{n_1 n_3}$.)

6.1. a. What are the energies (in joules and in eV) of photons having wavelengths of 820 nm and 1.3 μm?

b. What are the values of the free-space propagation constants for these wavelengths?

Solution: We are given that $\lambda_1 = 820 \times 10^{-9}$ and $\lambda_2 = 1.3 \times 10^{-6}$ m.

a. We find the photon energy E as

$$\begin{aligned} E &= h\nu = \frac{hc}{\lambda} \\ E_1 &= \frac{(6.63 \times 10^{-34})(3.0 \times 10^8)}{820 \times 10^{-9}} \\ &= 2.42 \times 10^{-19} \text{ joules} \Rightarrow 1.514 \text{ eV}\,. \end{aligned} \tag{8.7}$$

For the second wavelength, we have

$$\begin{aligned} E_2 &= \frac{(6.63 \times 10^{-34})(3.0 \times 10^8)}{1.3 \times 10^{-6}} \\ &= 1.529 \times 10^{-19} \text{ joules} \Rightarrow 0.956 \text{ eV}. \end{aligned} \tag{8.8}$$

b. We find the free-space propagation constant $k = 2\pi/\lambda$ as

$$k_1 = \frac{2\pi}{\lambda_1} = \frac{2\pi}{820 \times 10^{-9}} = 7.66 \times 10^6 \text{ m}^{-1}, \tag{8.9}$$

and

$$k_2 = \frac{2\pi}{\lambda_2} = \frac{2\pi}{1.3 \times 10^{-6}} = 4.83 \times 10^6 \text{ m}^{-1}. \tag{8.10}$$

6.3. Consider the absorption coefficient for silicon as shown in Figure 6.3 on page 165, repeated here as Fig. 8.1 on the following page. Ignoring surface reflections, plot the quantum efficiency for depletion layer widths of 1, 5, 10, 20, and 50 μm over the wavelength range of 0.6 to 1.0 μm. Assume that $d = 20$ μm.

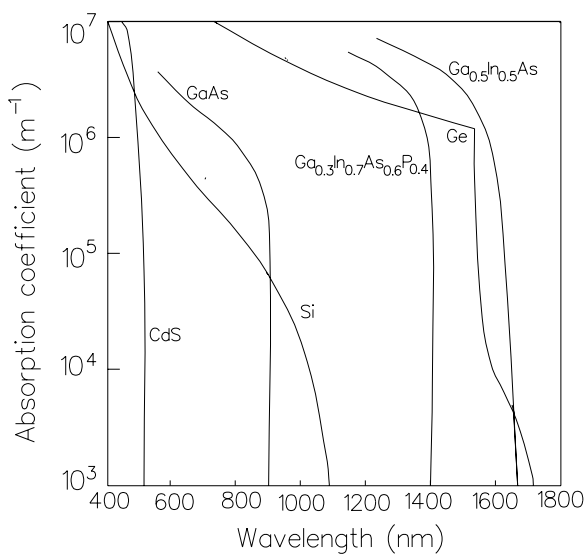

Figure 8.1: Absorption coefficient data.

Solution: We know that

$$P(w) = P_0(1 - R_f)e^{-\alpha d}(1 - e^{-\alpha w}). \tag{8.11}$$

If every absorbed photon contributes to the current flow

$$\begin{aligned} \eta &= \frac{P_{\text{absorbed}}}{P_{\text{incident}}} = \frac{P(w)}{P_0} = (1-R_f)e^{-\alpha d}(1-e^{-\alpha w}) \\ &= e^{-\alpha d}(1-e^{-\alpha w}) \quad \text{(ignoring reflections)}\,. \end{aligned} \tag{8.12}$$

I used a spreadsheet with coarsely sampled values of α to calculate the results. The resulting plot is shown in Fig. 8.2 on page 55. We note that thicker absorption regions are desirable, as is operation in the 850 to 950 nm range for silicon detectors.

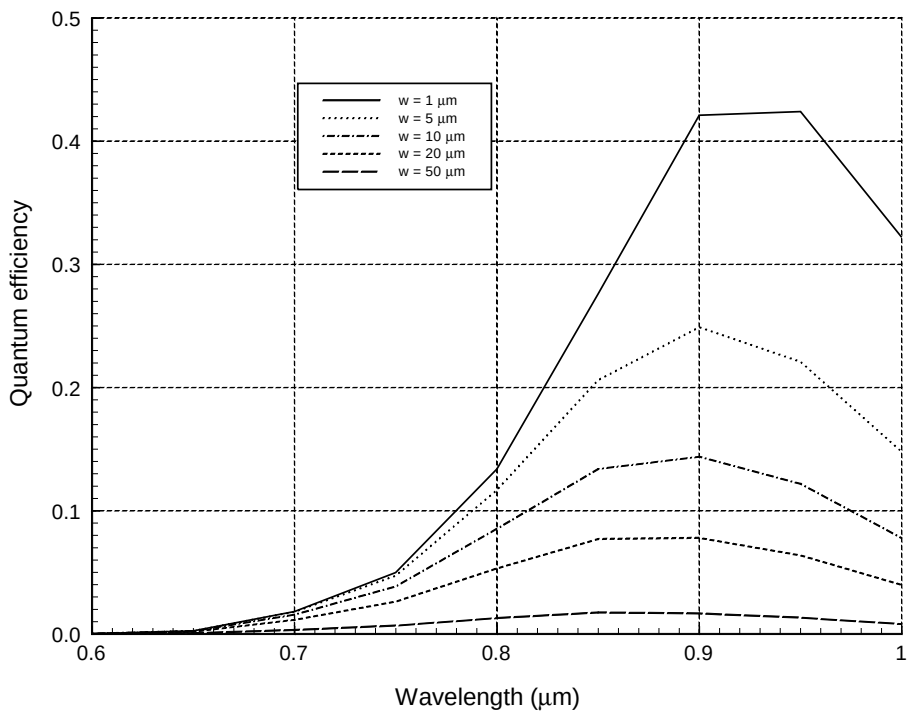

Figure 8.2: Results for Prob. 6.3. Quantum efficiency vs. wavelength for different absorption thicknesses in silicon.

6.4. An avalanche photodiode has the following parameters:

- dark current= 1 nA
- surface leakage current= 1 nA
- quantum efficiency= 0.85

- gain= 100
- excess noise factor= $M^{1/2}$
- load resistance= $10^4\Omega$

Consider a sinusoidal modulation on a 850 nm carrier with a modulation index m of 0.85. The average power level is -50 dBm and the detector operates at room temperature. Calculate the following S/N in dB if the bandwidth is 10 KHz.

a. The S/N if the signal-dependent shot noise is the dominant term. (Recall that $< i_s^2 >= (mM\mathcal{R}_0P)^2/2$.)

b. The S/N if the shot noise due to the dark current is the dominant term.

c. The S/N if the shot noise due to the surface current is the dominant noise term.

d. The S/N if the thermal noise is the dominant noise term.

e. Which is really the dominant noise source? What will the actual S/N be?

Solution: We are given that $I_d = 1 \times 10^{-9}$ A, $I_s = 1 \times 10^{-9}$ A, $\eta = 0.85$, $M = 100$, $F(M) = M^{0.5} = 10$, and $R_L = 1 \times 10^4\ \Omega$. Also that $\lambda = 850 \times 10^{-9}$ m, $m = 0.85$, $P = -50$ dBm $= 10 \times 10^{-9}$ W, $B = 1 \times 10^4$, and $T = 300$K.

a. If the shot noise is dominant, we begin by finding the $< i_s^2 >$. First, we need to find $\mathcal{R}_0$.

$$\mathcal{R}_0 = \frac{\eta q}{h\nu} = \frac{\eta q \lambda}{hc} = \frac{(0.85)(1.6 \times 10^{-19})(850 \times 10^{-9})}{(6.63 \times 10^{-34})(3.0 \times 10^8)} = 0.583 \text{ A/W}. \tag{8.13}$$

The mean-square signal current is

$$< i_s^2 >= \frac{m^2}{2}\mathcal{R}_0^2 P^2 M^2 = \frac{\left((0.85)(0.583)(1 \times 10^{-8})(100)\right)^2}{2} = 1.228 \times 10^{-13} \text{ A}^2. \tag{8.14}$$

The mean-square shot noise is

$$\begin{aligned} < i_N^2 > &= 2q\mathcal{R}_0 PBM^2F(M) \\ &= (2)(1.6 \times 10^{-19})(0.583)(1 \times 10^{-8})(1 \times 10^4)(1 \times 10^2)^2(10) \\ &= 1.866 \times 10^{-18} \text{ A}^2. \end{aligned} \tag{8.15}$$

The signal-to-noise ratio is, then,

$$\frac{S}{N} = \frac{< i_s^2 >}{< i_N^2 >} = \frac{1.228 \times 10^{-13}}{1.866 \times 10^{-18}} = 6.58 \times 10^4 \Rightarrow 48.2 \text{ dB}. \tag{8.16}$$

b. If we assume that the shot noise due to the dark current is dominant, we have

$$\begin{aligned} < i_N^2 > &= 2qI_D BM^2 F(M) \\ &= (2)(1.6 \times 10^{-19})(1 \times 10^{-9})(1 \times 10^4)(1 \times 10^2)^2(10) \\ &= 3.2 \times 10^{-19}\ \mathrm{A}^2\,, \end{aligned} \tag{8.17}$$

and the signal-to-noise ratio is found as

$$\frac{S}{N} = \frac{< i_s^2 >}{< i_N^2 >} = \frac{1.228 \times 10^{-13}}{3.2 \times 10^{-19}} = 3.84 \times 10^5 \Rightarrow 55.8\ \mathrm{dB}\,. \tag{8.18}$$

c. If we assume that the shot noise due to the surface dark current is dominant, we have

$$< i_N^2 >= 2qI_S B = (2)(1.6 \times 10^{-19})(1 \times 10^{-9})(1 \times 10^4) = 3.2 \times 10^{-24}\ \mathrm{A}^2\,, \tag{8.19}$$

and the signal-to-noise ratio is found as

$$\frac{S}{N} = \frac{< i_s^2 >}{< i_N^2 >} = \frac{1.228 \times 10^{-13}}{3.2 \times 10^{-24}} = 3.84 \times 10^{10} \Rightarrow 105.8\ \mathrm{dB}\,. \tag{8.20}$$

d. If we assume that the thermal noise is dominant, we have

$$< i_N^2 >= \frac{4kTB}{R_L} = \frac{(4)(1.38 \times 10^{-23})(300)(1 \times 10^4)}{1 \times 10^4} = 1.656 \times 10^{-20}\ \mathrm{A}^2\,, \tag{8.21}$$

and the signal-to-noise ratio is found as

$$\frac{S}{N} = \frac{< i_s^2 >}{< i_N^2 >} = \frac{1.228 \times 10^{-13}}{1.656 \times 10^{-20}} = 7.42 \times 10^6 \Rightarrow 68.7\ \mathrm{dB}\,. \tag{8.22}$$

e. We summarize the results in the table below.

Dominant noise	S/N
Signal shot	6.58×10^4
Bulk dark-current shot	3.84×10^5
Surface dark-current shot	3.84×10^{10}
Thermal	7.42×10^6

From an inspection of the results we see that the actual limiting noise source (i.e., the noise source that produces the *lowest* signal-to-noise ratio) is the shot noise produced by the signal and the resulting signal-to-noise ratio with *all* noise sources present is 6.58×10^4.