Chapter 6

Assignment 6 Solutions

5.1. Find ...

a. ...the minimum wavelength of a GaAlAs source.

b. ...the maximum wavelength of a GaAlAs source.

Solution: We know that, for $0 < x < 0.37$,

$$E'_g = 1.424 + 1.266x + 0.266x^2\,, \tag{6.1}$$

where E'_g is the bandwidth energy in eV and also that

$$\lambda' = \frac{1.240}{E'_g}\,, \tag{6.2}$$

where λ' is the wavelength in μm.

a) For $x = 0.37$,

$$\begin{aligned} E'_g &= 1.424 + 1.266x + 0.266x^2 \\ &= 1.424 + (1.266)(0.37) + 0.266(0.37)^2 = 1.929 \text{ eV} \end{aligned} \tag{6.3}$$

and

$$\lambda' = \frac{1.240}{E'_g} = \frac{1.240}{1.929} = 0.643\ \mu\text{m} = 643\ \text{nm}\,. \tag{6.4}$$

b) For $x = 0.0$,

$$\begin{aligned} E'_g &= 1.424 + 1.266x + 0.266x^2 \\ &= 1.424 + (1.266)(0) + 0.266(0)^2 = 1.424 \text{ eV} \end{aligned} \tag{6.5}$$

and

$$\lambda' = \frac{1.240}{E'_g} = \frac{1.240}{1.424} = 0.871\ \mu\text{m} = 871\ \text{nm}\,. \tag{6.6}$$

5.2. Using the alloy fraction formulas, find the material composition for ...

a. ... a 1.3 μm source.

b. ... a 1.55 μm source.

Solution: For an InP substrate we know that

$$E_g' = 1.35 - 0.72y + 0.12y^2\,, \tag{6.7}$$

where E_g' is the bandwidth energy in eV, and also that

$$\lambda' = \frac{1.240}{E_g'}\,, \tag{6.8}$$

where λ' is the wavelength *in μm*.

a) For $\lambda' = 1.300$,

$$E_g' = \frac{1.240}{\lambda'} = \frac{1.240}{1.300} = 0.954\,. \tag{6.9}$$

Substituting, we have

$$\begin{aligned} E_g' &= 1.35 - 0.72y + 0.12y^2 \\ 0.954 &= 1.35 - 0.72y + 0.12y^2 \\ 0 &= 0.12y^2 - 0.72y + 0.396\,. \end{aligned} \tag{6.10}$$

Solving the quadratic formula, we obtain two solutions for y. We discard the solution that is greater than 1, and find

$$\begin{aligned} y &= 0.613 \\ 1-y &= 0.387\,. \end{aligned} \tag{6.11}$$

We find x from

$$x = \frac{0.4526}{1-0.031y} = \frac{0.4526}{1-(0.031)(0.613)} = 0.469 \tag{6.12}$$

and, hence,

$$1 - x = 0.531\,. \tag{6.13}$$

So the alloy is $In_{0.469}Ga_{0.535}As_{0.613}P_{0.387}$ for a 1300 nm source.

b) For $\lambda' = 1.550$,

$$E_g' = \frac{1.240}{\lambda'} = \frac{1.240}{1.550} = 0.800\,. \tag{6.14}$$

Substituting, we have

$$\begin{aligned} E_g' &= 1.35 - 0.72y + 0.12y^2 \\ 0.800 &= 1.35 - 0.72y + 0.12y^2 \\ 0 &= 0.12y^2 - 0.72y + 0.550\,. \end{aligned} \tag{6.15}$$

Solving the quadratic formula, we obtain two solutions for y. We discard the solution that is greater than 1, and find

$$\begin{aligned} y &= 0.898 \\ 1-y &= 0.1016\,. \end{aligned} \tag{6.16}$$

We find x from

$$x = \frac{0.4526}{1-0.031y} = \frac{0.4526}{1-(0.031)(0.898)} = 0.469 \tag{6.17}$$

and, hence,

$$1-x = 0.531\,. \tag{6.18}$$

So the alloy is $In_{0.469}Ga_{0.531}As_{0.898}P_{0.1616}$ for a 1550 nm source.

5.3. Consider two $Ga_{1-x}Al_xAs$ laser sources with $x = 0.02$ for the first and $x = 0.09$ for the other. Find the bandgap energy and peak wavelength for these devices.

Solution: We know that for $0 < x < 0.37$,

$$E_g' = 1.424 + 1.266x + 0.266x^2\,, \tag{6.19}$$

where E_g' is the bandwidth energy in eV and also that

$$\lambda' = \frac{1.240}{E_g'}\,, \tag{6.20}$$

where λ' is the wavelength in μm.

a) For $x = 0.02$,

$$\begin{aligned} E_g' &= 1.424 + 1.266x + 0.266x^2 \\ &= 1.424 + (1.266)(0.02) + 0.266(0.02)^2 = 1.449 \text{ eV} \end{aligned} \tag{6.21}$$

and

$$\lambda' = \frac{1.240}{E_g'} = \frac{1.240}{1.449} = 0.855\ \mu\text{m} = 855\text{ nm}\,. \tag{6.22}$$

b) For $x = 0.09$,

$$\begin{aligned} E_g' &= 1.424 + 1.266x + 0.266x^2 \\ &= 1.424 + (1.266)(0.09) + 0.266(00.09)^2 = 1.540 \text{ eV} \end{aligned} \tag{6.23}$$

and

$$\lambda' = \frac{1.240}{E_g'} = \frac{1.240}{1.540} = 0.805\ \mu\text{m} = 805\text{ nm}\,. \tag{6.24}$$

5.4. We want to estimate the transmission at air–material interfaces for high refractive–index materials like InP and GaAs. For a plane wave at perpendicular incidence, the transmissivity of a planar interface between a material and air ($n = 1$) is

$$T = 1 - \left(\frac{n-1}{n+1}\right)^2 , \tag{6.25}$$

where n is the index of refraction of the material. Find T in percent and in dB for ...

a. ... InP ($n = 3.4$).

b. ... GaAs ($n = 3.6$).

<u>Solution:</u> a) For $n = 3.4$, we have

$$T = 1 - \left(\frac{n-1}{n+1}\right)^2 = 1 - \left(\frac{3.4-1}{3.4+1}\right)^2 = 0.702 \Rightarrow -1.534 \text{ dB} \tag{6.26}$$

or a transmission of 70.2%.

b) For $n = 3.6$, we have

$$T = 1 - \left(\frac{n-1}{n+1}\right)^2 = 1 - \left(\frac{3.6-1}{3.6+1}\right)^2 = 0.681 \Rightarrow -1.672 \text{ dB} \tag{6.27}$$

or a transmission of 68.1%.

5.5. Show that the LED bandwidth is given by Eq. 5.20 on page 121.

Solution: We begin with the equation for the optical power

$$P_{\text{out}} = \frac{P_0}{1 + 4\pi^2 f^2 \tau_{\text{lifetime}}^2} . \tag{6.28}$$

We want to find the frequency $f_{3-\text{dB}}$ that makes $P_{\text{out}} = P_0/2$, so

$$\begin{aligned} \frac{P_{\text{out}}}{P_0} &= \frac{1}{2} = \frac{1}{1 + 4\pi^2 f_{3-\text{dB}}^2 \tau_{\text{lifetime}}^2} \\ 2 &= 1 + 4\pi^2 f_{3-\text{dB}}^2 \tau_{\text{lifetime}}^2 \\ 1 &= 4\pi^2 f_{3-\text{dB}}^2 \tau_{\text{lifetime}}^2 \\ f_{3-\text{dB}}^2 &= \frac{1}{4\pi^2 \tau_{\text{lifetime}}^2} \\ f_{3-\text{dB}} &= \frac{1}{2\pi \tau_{\text{lifetime}}} . \end{aligned} \tag{6.29}$$

5.6 An optical source is selected from group of devices specified as requiring a mean time of 5×10^4 hours for the output power to degrade by -3 dB. If the device emits 5 mW at room temperature at the start of a test, what will be its emission power after (a) 1 month, (b) 1 year, (c) 5 years and (d) 10 years?

Solution: The long–term power behavior of a laser is

$$P = P_0 e^{(-t/\tau_m)} . \tag{6.30}$$

We begin by calculating τ_m, from

$$\begin{aligned} P &= P_0 e^{(-t/\tau_m)} \\ \frac{P}{P_0} &= 0.5 = e^{\left(-5\times10^4/\tau_m\right)} \\ \tau_m &= \frac{5\times10^4}{\ln(0.5)} = 7.21\times10^4 \text{ hrs} . \end{aligned} \tag{6.31}$$

a. For $t = 1$ month $= 1(30)(24) = 720$ hours,

$$P = P_0 e^{(-t/\tau_m)} = 5e^{\left(-720/(7.21\times10^4)\right)} = 4.95 \text{ mW} . \tag{6.32}$$

b. For $t = 1$ year $= (1)(365)(24) = 8.76 \times 10^3$ hours,

$$P = P_0 e^{(-t/\tau_m)} = 5e^{\left(-(8.76\times10^3)/(7.21\times10^4)\right)} = 4.43 \text{ mW} . \tag{6.33}$$

c. For $t = 5$ years $= (5)(365)(24) = 4.38 \times 10^4$ hours,

$$P = P_0 e^{(-t/\tau_m)} = 5e^{\left(-(4.38\times10^4)/(7.21\times10^4)\right)} = 2.72 \text{ mW} . \tag{6.34}$$

d. For $t = 10$ years $= (10)(365)(24) = 8.76 \times 10^4$ hours,

$$P = P_0 e^{(-t/\tau_m)} = 5e^{\left(-(8.76\times10^4)/(7.21\times10^4)\right)} = 1.484 \text{ mW} . \tag{6.35}$$

5.7. A group of laser devices has an operating lifetime of 3.5×10^4 at 60C and 6700 hours at 90C. Find the activation energy for these devices and calculate the expected lifetime at 20C.

Solution: We have $\tau_1 = 3.5 \times 10^4$ hours for $T = 60\text{C} = 330\text{K}$ and $\tau_2 = 6.7 \times 10^3$ hrs for $T = 90\text{C} = 360\text{K}$.

To find the activation energy E, we find

$$\begin{aligned} \frac{\tau_1}{\tau_2} &= \frac{e^{\frac{E}{kT_1}}}{e^{\frac{E}{kT_2}}} \\ &= e^{\frac{E}{k}\left(\frac{1}{T_1}-\frac{1}{T_2}\right)} \end{aligned} \tag{6.36}$$

$$\begin{aligned}
\ln\left(\frac{\tau_1}{\tau_2}\right) &= \frac{E}{k}\left(\frac{1}{T_1}-\frac{1}{T_2}\right) && (6.37)\\
E &= k\ln\left(\frac{\tau_1}{\tau_2}\right)\frac{1}{\left(\frac{1}{T_1}-\frac{1}{T_2}\right)} && (6.38)\\
&= (1.38\times10^{-23})\ln\left(\frac{3.5\times10^4}{6.7\times10^3}\right)\frac{1}{\left(\frac{1}{330}-\frac{1}{360}\right)}\\
&= 9.05\times10^{-20}\text{ joules}\Rightarrow 0.566\text{ eV}.
\end{aligned}$$

We want to calculate the expected lifetime at $T = 20\text{C} = 290\text{K}$. We find this from

$$\begin{aligned}
\frac{\tau_1}{\tau_3} &= \exp\left(\frac{E}{k}\left(\frac{1}{T_1}-\frac{1}{T_3}\right)\right) && (6.39)\\
\tau_3 &= \frac{\tau_1}{\exp}\left(\frac{E}{k}\left(\frac{1}{T_1}-\frac{1}{T_3}\right)\right)\\
&= \frac{3.5\times10^4}{\exp}\left(\frac{9.05\times10^{-20}}{1.38\times10^{-23}}\left(\frac{1}{330}-\frac{1}{290}\right)\right) = 5.43\times10^5\text{ hrs}.
\end{aligned}$$

5.8. A laser diode has a lateral beam divergence of 30 degrees (full angle) and a perpendicular (to the emitting junction) beam divergence of 60 degrees. What are the values of L and T associated with this beam pattern?

Solution: The asymmetric beam pattern is given by

$$B(\theta,\phi) = \frac{1}{\left(\frac{\sin^2\phi}{B_0\cos^T\theta}+\frac{\cos^2\phi}{B_0\cos^L\theta}\right)}. \qquad (6.40)$$

We want to find T and L.

We observe that, when $\phi = 0$, $B(\theta, 0) = B_0\cos^L\theta$. From the definition of beam divergence, we have

$$\begin{aligned}
\frac{B(\pm15^\circ, 0^\circ)}{B_0} &= \frac{1}{2} && (6.41)\\
\cos^L 15^\circ &= \frac{1}{2}\\
(0.966)^L &= 0.5\\
L\ln(0.966) &= \ln(0.5)\\
L &= \frac{\ln(0.5)}{\ln(0.966)} = 20.03
\end{aligned}$$

We also observe that, when $\phi = 90°$, $B(\theta, 90°) = B_0 \cos^T \theta$, and

$$\begin{aligned}
\frac{B(\pm 30°, 90°)}{B_0} &= \frac{1}{2} \qquad (6.42)\\
\cos^T 30° &= \frac{1}{2}\\
(0.866)^T &= 0.5\\
T \ln(0.866) &= \ln(0.5)\\
T &= \frac{\ln(0.5)}{\ln(0.866)} = 4.81
\end{aligned}$$