EC 3210 Solutions

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Assignment 8

8.1. Calculate the mirror reflectance required to sustain lasing if the gain coefficient is 1.2 m^{-1}. The laser resonator length is 10 cm and the reflectance of both mirrors is assumed to be the same. (Assume that the internal losses are negligible.)

We have $\beta = 1.2\ \mathrm{m}^{-1}$, $L = 10\ \mathrm{cm} = 0.1\ \mathrm{m}$, and $R_1 = R_2 = R$. Also,$\alpha_{\mathrm{int}} = 0$. Since the device is lasing, the gain coefiicient equals the threshold gain coefficient.

$$\beta = \beta_{\mathrm{th}} = \alpha_{\mathrm{int}} + \frac{1}{2L}\ln\left(\frac{1}{R^2}\right) \tag{1a}$$

$$1.2 = 0 + \frac{1}{2(0.1)}\ln\left(\frac{1}{R^2}\right) \tag{1b}$$

$$\ln\left(\frac{1}{R^2}\right) = 1.2(2)(0.1) = 2.40 \times 10^{-1} \tag{1c}$$

$$\frac{1}{R^2} = 1.271 \tag{1d}$$

$$R^2 = 0.786 \tag{1e}$$

$$R = 0.887 \Rightarrow 88.7\%\,. \tag{1f}$$

8.4. Consider a Neodymium laser rod that is 15 cm long and 1 cm in diameter. The material has a Lorentzian lineshape with a linewidth of 10 GHz.

a. Calculate the minimum pump power that would be necessary to achieve a threshold population inversion in Nd of 9×10^{15} atoms. The upper energy state is 1.4 eV above the ground state and the spontaneous lifetime is 300 μs.

b. Consider the result of the previous section. Suppose

- that the flashlamp source has an efficiency of 10% in converting electricity to light,
- that 10% of the light produced by the flashlamp falls on the Nd impregnated crystal, and
- that 5% of the incident light is absorbed by the crystal.

Calculate the minimum electric power required to be applied to the flashlamp.

a. We have the energy level diagram shown in Fig. 1.

Neodymium is a four-level laser material. We have a population difference of $\Delta N = 9 \times 10^{15}$ atoms and a spontaneous lifetime of the upper lasing level of $\tau_s = 300 \times 10^{-6}$ s.

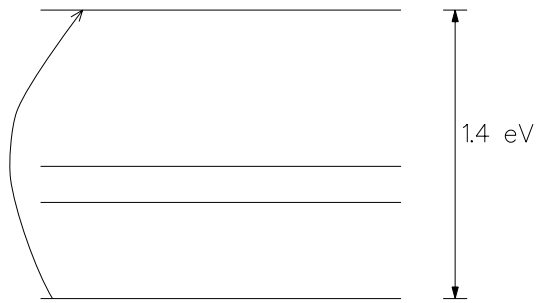

Figure 1: Energy level diagram for Problem 8.4a.

The pump energy required to pump up an atom is $E = \Delta E$, so the minimum pump energy to pump up ΔN atoms is $E_{\min} = \Delta N\, \Delta E$. We must pump the atoms in a time equal or less than τ_s, so

$$P_{\min} = \frac{E_{\min}}{\tau_s} = \frac{\Delta N\, \Delta E}{\tau_s} = \frac{(9 \times 10^{15})(1.4 \times 1.6 \times 10^{-19})}{300 \times 10^{-6}} = 6.72 \text{ W}\,. \tag{2}$$

b. We want to find the total pump power required in the face of various pump ineficiencies.

$$P_{\min \text{ pump}} = \frac{P_{\min}}{\eta_1\, \eta_2\, \eta_3} = \frac{6.72}{(0.1)(0.1)(0.05)} = 1.344 \times 10^4 \text{ W}\,. \tag{3}$$

8.5. Consider a four-level lasing material that is 10 cm long and has a cross-sectional area 1 cm^2. The energy gaps are: $E_{01} = 1.5$ eV, $E_{12} = 2.5$ eV, and $E_{23} = 1.0$ eV, The pertinent lifetimes are: $\tau_{32} = 1$ ns, $\tau_{21\ \text{spon}} = 1$ ms, and $\tau_{10} = 1$ ns. The index of refraction of the material is 1.

The material is homogeneously broadened with a resulting Lorentzian lineshape and has a 1 GHz linewidth. The internal loss coefficient α_{int} is assumed to be 0. The material is placed between mirrors (each with a 30 cm radius of curvature) that are 10 cm apart. The mirror reflectivities are 100% and 95%.

Calculate:

a. the nominal laser wavelength

b. the nominal lasing frequency

c. the wavelength of the pump (assuming optical pumping)

d. the frequency of the pump

e. if the laser resonator is stable

f. the threshold gain coefficient

g. the threshold population density $(N_2 - N_1)_{th}/\text{Vol}$

h. the threshold population inversion

i. the threshold pump rate

j. the minimum threshold pump power

Suppose that we pump the laser at five times the threshold pump rate, calculate:

k. the saturation irradiance, I_s

l. the irradiance inside the laser resonator

m. the output irradiance

n. the output power of the laser

o. the unsaturated population inversion density

p. the unsaturated population inversion

q. the unsaturated gain coefficient of the lasing medium

The energy diagram of the material is given in Fig. **??**. We are also given that $\tau_{32} = 1 \times 10^{-9}$, $\tau_{21\ sp} = 1 \times 10^{-3}$, and $\tau_{10} = 1 \times 10^{-9}$ seconds.

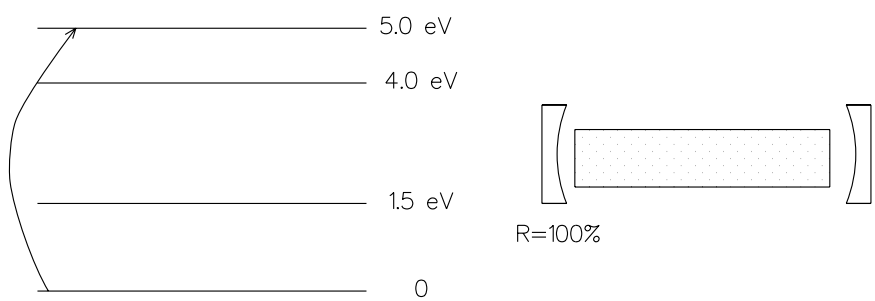

Figure 2: Energy level diagram and resonator for Problem 8.5.

a. The lasing wavelength is

$$\lambda = \frac{hc}{\Delta E_{21}} = \frac{(6.63 \times 10^{-34})(3.0 \times 10^{8})}{(2.5)(1.6 \times 10^{-19})} = 498 \times 10^{-9} = 498 \text{ nm}\,. \tag{4}$$

b. The lasing frequency is

$$\nu = \frac{\Delta E_{21}}{h} = \frac{(2.5)(1.6 \times 10^{-19})}{6.63 \times 10^{-34}} = 6.02 \times 10^{14} \text{ Hz}\,. \tag{5}$$

c. The pump wavelength (assuming optical pumping) is

$$\lambda_{\text{pump}} = \frac{hc}{\Delta E_{30}} = \frac{(6.63 \times 10^{-34})(3.0 \times 10^{8})}{(5.0)(1.6 \times 10^{-19})} = 249 \times 10^{-9} \text{ m} = 249 \text{ nm}\,.$$

(This is an ultraviolet wavelength.)

d. The pump frequency is

$$\nu = \frac{\Delta E_{30}}{h} = \frac{(5.0)(1.6 \times 10^{-19})}{6.63 \times 10^{-34}} = 12.05 \times 10^{14} \text{ Hz}\,. \tag{6}$$

e. Is the resonator stable?

$$g_1 = g_2 = 1 - \frac{L}{r_1} = 1 - \frac{10}{30} = 0.667 \tag{7a}$$

so

$$g_1 g_2 = 0.444\,. \tag{7b}$$

Since $g_1 g_2$ is between 0 and 1, the resonator is stable.

f. The threshold gain coefficient β_{th} is

$$\beta_{\text{th}} = \alpha_{\text{int}} + \frac{1}{2L} \ln\left(\frac{1}{R_1 R_2}\right) = 0 + \frac{1}{0.2} \ln\left(\frac{1}{(1)(0.95)}\right) = 0.256 \text{ m}^{-1}\,. \tag{8}$$

g. The threshold population inversion density $\Delta N_{th}/\text{Vol}$ is given by

$$\frac{(N_2 - N_1)_{\text{th}}}{\text{Vol}} = \frac{\lambda \beta_{\text{th}}}{B_{21} h g(\nu_0)}\,. \tag{9}$$

First we need to do some intermediate calculations:

$$g(\nu_0) = \frac{2}{\pi\,\Delta\nu} = \frac{2}{\pi(1 \times 10^{9}}\text{s} = 6.37 \times 10^{-10}\,. \tag{10a}$$

$$A_{21} = \frac{1}{\tau_{21\ sp}} = \frac{1}{1 \times 10^{-3}} = 1 \times 10^{3}\,. \tag{10b}$$

and

$$B_{21} = \frac{\lambda^3 A_{21}}{8\pi h} = \frac{(498 \times 10^{-9})^3 (1 \times 10^{3})}{8\pi(6.63 \times 10^{-34})} = 7.40 \times 10^{15}\,. \tag{10c}$$

So,

$$\begin{aligned} \frac{(N_2 - N_1)_{\text{th}}}{\text{Vol}} &= \frac{\lambda \beta_{\text{th}}}{B_{21} h g(\nu_0)} = \frac{(0.256)(497 \times 10^{-9})}{(7.37 \times 10^{15})(6.63 \times 10^{-34})(6.36 \times 10^{-10})} \\ &= 4.08 \times 10^{19} \text{ atoms} \cdot \text{m}^{-3}\,. \end{aligned} \tag{11}$$

h. We now want to find the threshold population inversion $(N_j - N_i)_{\text{th}}$, so we have

$$(N_2 - N_1)_{\text{th}} = 4.08 \times 10^{19} \times \text{Vol} \tag{12a}$$

$$\text{Vol} = LA = (0.1)(1 \times 10^{-4}) = 1 \times 10^{-5}\ \text{m}^3\,. \tag{12b}$$

$$(N_2 - N_1)_{\text{th}} = (4.08 \times 10^{19})(1 \times 10^{-5}) = 4.08 \times 10^{14}\ \text{atoms}\,. \tag{12c}$$

i. The threshold pump rate R_{th} is

$$R_{\text{th}} = \frac{A_{21}(N_2 - N_1)_{\text{th}}}{(1 - \frac{A_{21}}{A_{10}})} = \frac{(1 \times 10^3)(4.08 \times 10^{14}}{1 - \frac{1\times 10^3}{1\times 10^9}} = 4.08 \times 10^{17}\ \text{atoms} \cdot \text{s}^{-1}\,. \tag{13}$$

j. The minimum threshold pump power $P_{\text{pump min}}$ is found from

$$P_{\text{pump min}} = R_{\text{th}} \times \Delta E_{30} = (4.08 \times 10^{17})(5)(1.6 \times 10^{19}) = 0.327\ \text{W}\,. \tag{14}$$

Suppose that we pump at 5x the minimum pump rate.

k. The saturation irradiance I_s is found by

$$\begin{aligned} I_s &= \frac{8\pi hc}{\lambda^3 g(\nu_0)} = \frac{8\pi(6.63 \times 10^{-34})(3.0 \times 10^8)}{(497 \times 10^{-9})^3(6.36 \times 10^{-10})} \\ &= 6.37 \times 10^4\ \text{W} \cdot \text{m}^{-2} = 6.37\ \text{W} \cdot \text{cm}^{-2}\,. \end{aligned} \tag{15}$$

l. The irradiance I inside the resonator is

$$I = I_s\left(\frac{R}{R_{\text{th}}} - 1\right) = I_s(5-1) = 4I_s = 2.55 \times 10^5\ \text{W} \cdot\ \text{m}^{-2} = 25.5\ \text{W} \cdot\ \text{cm}^{-2}\,. \tag{16}$$

m. The output irradiance I_{out} is

$$I_{\text{out}} = T_2 I = (0.05)(2.55 \times 10^5) = 1.274 \times 10^4\ \text{W} \cdot\ \text{m}^{-2} = 1.274\ \text{W} \cdot \text{cm}^{-2}\,. \tag{17}$$

n. The output power P_{out} is

$$P_{\text{out}} = AI_{\text{out}} = (1 \times 10^{-4})(1.274 \times 10^4) = 1.274\ \text{W}\,. \tag{18}$$

o. The unsaturated population inversion density $\Delta N_0/\text{Vol}$ is

$$\begin{aligned} \frac{(N_2 - N_1)_0}{\text{Vol}} &= 5 \times \frac{(N_2 - N_1)_{\text{th}}}{\text{Vol}} \\ &= (5)(4.10 \times 10^{19}) = 20.4 \times 10^{19}\ \text{atoms} \cdot \text{m}^{-3}\,. \end{aligned} \tag{19}$$

p. The unsaturated population inversion ΔN_0 is

$$(N_2 - N_1)_0 = 5 \times (N_2 - N_1)_{\text{th}} = (5)(4.10 \times 10^{14}) = 20.4 \times 10^{14}\ \text{atoms}\,. \tag{20}$$

q. The unsaturated gain coefficient β_0 is

$$\beta_0 = 5 \times \beta_{\text{th}} = (5)(0.256) = 1.282 \text{ m}^{-1} . \tag{21}$$

12.4. The lower level of a Nd:YAG laser is 2111 cm^{-1} above ground level. It is asserted that this level is more than "several factors" of kT at room temperature ($T = 300$K). Calculate the ratio of ΔE to kT.

We are given that the lower energy level of the Nd:YAG laser is 2,111 cm^{-1} above the ground state. We want to show that $\Delta E/kT \ll 1$. We find

$$\frac{\Delta E}{kT} = \frac{(2111)(1.99 \times 10^{-23})}{(1.38 \times 10^{-23})(300)} = 10.1 . \tag{22}$$

So, we see that the ratio is about 10:1.

12.5. Consider a diode laser that has an emitting area that is 3 μm by 0.1 μm and emits 100 mW of power. If the output mirror is 80% reflecting, calculate the irradiance level (in W·cm^2) inside the cavity.

We are given a diode laser with an emitting area that measures 3 μm by 0.1 μm that emits 100 mW. We begin by calculating the irradiance just outside the laser emitting face.

$$I_{\text{outside}} = \frac{P}{A} = \frac{100 \times 10^{-3}}{(3 \times 10^{-6})(0.1 \times 10^{-6})} = 3.33 \times 10^{11} \text{ W} \cdot \text{m}^{-2} . \tag{23}$$

We find the irradiance level I inside the cavity from $I_{\text{outside}} = T\, I_{\text{inside}}$, so we have

$$I_{\text{inside}} = \frac{I_{\text{outside}}}{T} = \frac{3.33 \times 10^{11}}{0.2} = 16.65 \times 10^{11} \text{ W} \cdot \text{m}^{-2} = 16.65 \times 10^{7} \text{ W} \cdot \text{cm}^{-2} . \tag{24}$$

We note that the irradiance inside is $1/T_2$ times that outside the resonator.