EC 3210 Solutions

Assignment 2

2.1. Consider a Young's dual slit experiment operated with a HeNe laser at 632.8 nm.
a) Calculate the slit spacing if the fringe spacing is 1.5 mm and the slit-to-screen distance is 2 meters.
b.) One slit is now covered with a piece of glass ($n = 1.5$) that is 1 mm thick. What will be the resulting movement in the fringes?

Solution: We have a Young's dual slit experiment with $\lambda = 632.8$ nm, $R = 2$ m, and $\Delta x = 1.5$ mm. We want to find the slit spacing D.

(a.) We first find θ,

$$\Delta x = \frac{\lambda}{\sin\theta} \tag{1}$$
$$\sin\theta = \frac{\lambda}{\Delta x} = \frac{632.8 \times 10^{-9}}{1.5 \times 10^{-3}} = 4.22 \times 10^{-4}$$
$$\theta = 4.22 \times 10^{-4} \text{ (by the small angle approximation)} = 0.0242^\circ .$$

and, hence,

$$\tan\theta = \frac{D}{R} \approx \theta \tag{2}$$
$$D \approx R\theta = (2)(4.22 \times 10^{-4}) = 8.44 \times 10^{-4} \text{ m} = 844 \quad \mu\text{m} .$$

(b.) The addition of the glass plate changes the phase of the path:

$$\Delta\phi = kL = \frac{2\pi}{\lambda}L = \frac{2\pi}{\frac{v}{\nu}}L = \frac{2\pi\nu}{v}L = \frac{2\pi\nu}{c/n}L = \frac{2\pi}{\lambda_0}nL \tag{3}$$

The *additional* phase through the glass instead of air is

$$\Delta\phi_2 - \Delta\phi_1 = \frac{2\pi}{\lambda_0}(n-1)L \tag{4}$$
$$= \frac{(2\pi)(1.5-1)(1 \times 10^{-3})}{632.8 \times 10^{-9}} = 2\pi(7.901 \times 10^2) .$$

Hence, there are 790.1 additional cycles of 2π phase. This will result in the motion of 790.1 fringes past the point detector.

2.2. Consider an Argon laser operating at 488 nm. Calculate the frequency linewidth and the spectral linewidth to ensure a coherence length of 1 m (as might typically be required to use the laser in holography applications).

Laser	Typical coherence length
single-mode HeNe	up to 1000 m
multimode HeNe	10 to 20 cm
multimode Argon	2 cm
Nd:YAG	1 cm
Nd:glass	0.2 mm
GaAs semiconductor	1 mm
Ruby (pulse train)	1 cm
Ruby (single pulse)	$\leq$ 30 m

Table 1: Representative coherence lengths of typical lasers

Solution: We have an argon laser $\lambda = 488 \times 10^{-9}$ m. We want $l_c = 1$ m, so

$$t_c = \frac{1}{\Delta\nu} = \frac{l_c}{c} = \frac{1}{3 \times 10^8} = 3.33 \text{ ns} \tag{5}$$

and

$$\Delta\nu = \frac{1}{t_c} = 3 \times 10^8 \text{ Hz} = 300 \text{ MHz}\,. \tag{6}$$

We find $\Delta\lambda$ from

$$\Delta\nu = \frac{c\,\Delta\lambda}{\lambda^2} \tag{7}$$

$$\Delta\lambda = \frac{\lambda^2\,\Delta\nu}{c} = \frac{(488 \times 10^{-9})^2 (3 \times 10^8)}{3.0 \times 10^8} = 2.38 \times 10^{-13} = 0.238 \text{ pm}\,.$$

2.3. Table 1 lists some typical coherence lengths of some representative lasers. Calculate the coherence times for these lasers.

Solution:

Laser	l_c	$t_c = l_c/c$ s
HeNe	1,000 m	3.33×10^{-6} s
Multimode HeNe	20 cm	6.67×10^{-10} s
Multimode Argon	2 cm	6.67×10^{-11} s
Nd:YAG	1 cm	3.33×10^{-11} s
Nd:glass	0.20 mm	6.67×10^{-13} s
GaAs	1 mm	3.33×10^{-12} s
Ruby (pulse train)	1.0 cm	3.33×10^{-11} s
Ruby (single pulse)	30 m	1.000×10^{-7} s

2.4 A laser has a longitudinal coherence length of 1 mm. If the spectral linewidth is 1 nm, find the frequency of the laser.

Solution: We are given that $l_c = 1$ mm and that $\Delta\lambda = 1$ nm and we want to find ν. We know that

$$t_c = \frac{1}{\Delta\nu} = \frac{l_c}{c} \tag{8}$$

$$\Delta\nu = \frac{c}{l_c} = \frac{3.0 \times 10^8}{1 \times 10^{-3}} = 3 \times 10^{11} \text{ Hz}.$$

We also know that

$$|\Delta\lambda| = \frac{\lambda^2}{c}|\Delta\nu| = \frac{\lambda^2}{c^2}c|\Delta\nu| = \frac{1}{\nu^2}c|\Delta\nu| \tag{9}$$

$$\nu^2 = \frac{c|\Delta\nu|}{|\Delta\lambda|}$$

$$\nu = \sqrt{\frac{c|\Delta\nu|}{|\Delta\lambda|}} = \sqrt{\frac{(3.0 \times 10^8)(3 \times 10^{11})}{1 \times 10^{-9}}} = 3 \times 10^{14} \text{ Hz}.$$

2.5. A Mach-Zender interferometer is adjustable in displacement d. If $\lambda = 1.06$ μm, $d = 25$ cm, and the distance between beamsplitter #1 and beamsplitter #2 is 20 cm,

a) . . . calculate the pathlength difference.

b.) . . . calculate the relative time delay introduced by the longer path.

c.) . . . calculate the total phase delay between plane waves taking the two paths.

The displacement d is increased to 30 cm. Recalculate the previous answers.

Solution: We adjust a Mach–Zender spacing d. We have $\lambda = 10.6$ μm and $d = 25$ cm, and the beamsplitter spacing is 20 cm. (See Fig. 1.)

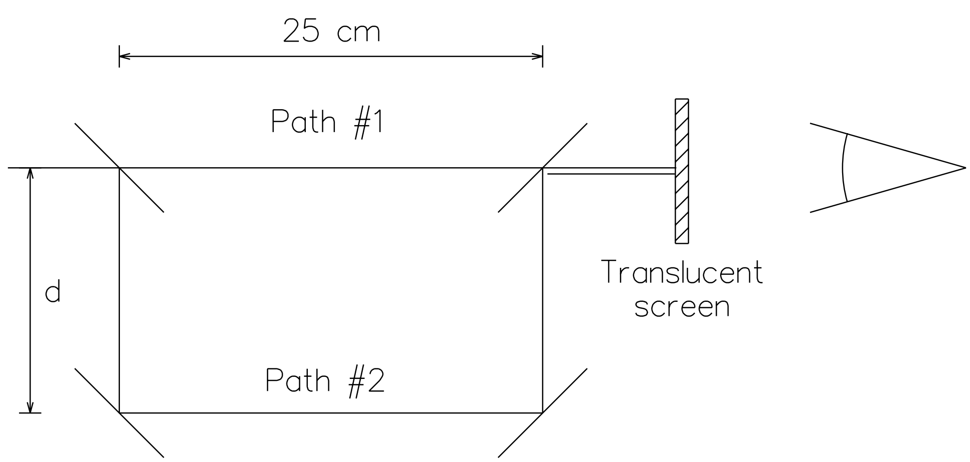

Figure 1: Problem 2.5. Mach-Zender geometry.

(a.) The path-length difference is

$$\Delta L = 2d = 2(25) = 50 \text{ cm}. \tag{10}$$

(b.) The time difference is

$$\Delta t = \frac{\Delta L}{c} = \frac{50 \times 10^{-2}}{3.0 \times 10^{8}} = 1.667 \times 10^{-9}\ \mathrm{s}\,. \tag{11}$$

(c.) The phase shift is

$$\Delta\phi = \frac{2\pi\,\Delta L}{\lambda} = \frac{2\pi(50 \times 10^{-2})}{1.06 \times 10^{-6}} = 2.96 \times 10^{6}\ \text{radians}\,. \tag{12}$$

If d increases to 30 cm, we recalculate

(a′.) $\Delta L = 2d' = 60$ cm.

(b′.) $\Delta t = \Delta L/c = 60 \times 10^{-2}/3.0 \times 10^{8} = 2 \times 10^{-9} = 2$ ns.

(c′.) $\Delta\phi = 2\pi\,\Delta L/\lambda = 2\pi(60 \times 10^{-2})/1.06 \times 10^{-6} = 3.56 \times 10^{6}$ radians.

2.6. One of the mirrors of the Michelson interferometer moves with a velocity v. Show that the rate at which the fringes would pass a point detector is $2v/\lambda$. This allows the use of the interferometer as a precise measurement of small velocities.

Solution: We have a moving mirror on a Michelson interferometer moving with a velocity v. The fringes will shift one Λ for every $\lambda/2$ movement of the mirror. The number of $\lambda/2$ that the mirror moves per second is

$$N = \frac{v}{\frac{\lambda}{2}}\,. \tag{13}$$

The frequency that the fringes move past a point detector is N per second or N Hz, hence

$$f_{\text{fringe}} = N = \frac{2v}{\lambda}\ \mathrm{Hz}\,. \tag{14}$$