III. Amplitude Modulation

AM schemes

- Double sideband suppressed carrier (DSB-SC)
- Double sideband transmitted carrier (DSB-WC)
- Modulation efficiency & index
- Single sideband (SSB)
- Vestigial sideband (VSB)
- Amplitude shift keying (ASK)

Modulators

- Gated modulator
- Square law modulator
- SSB modulator
- Vestigial modulator

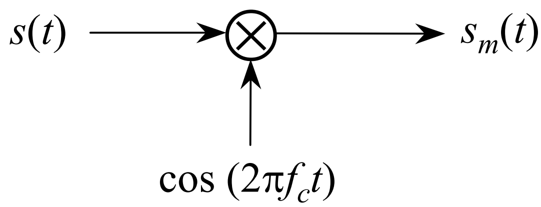
Demodulators

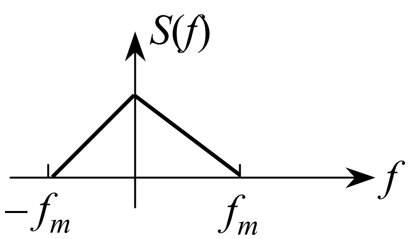
- Coherent demodulation
  - gated demodulation
  - frequency mismatch effects
  - quadrature receiver
  - SSB demodulation
  - VSB demodulation
  - ASK demodulation
- Incoherent demodulation
- rectifier detector
- envelope detector
- SSB incoherent demodulation
- ASK incoherent demodulation

AM Broadcast

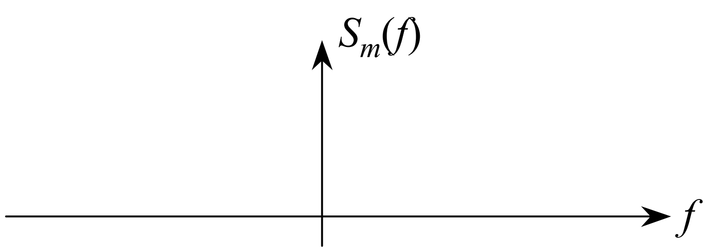
III. Amplitude Modulation

1) Double Sideband Suppressed Carrier

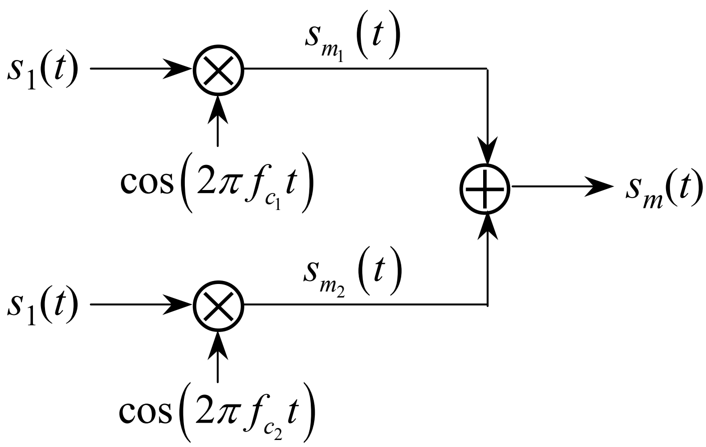


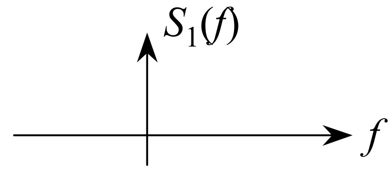


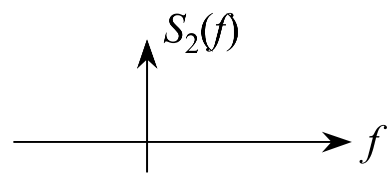
$S_m(f) =$

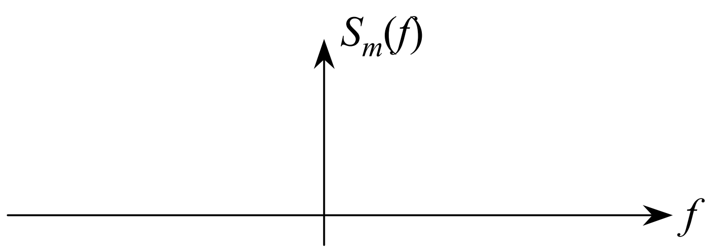


- Multiple Signal Transmission



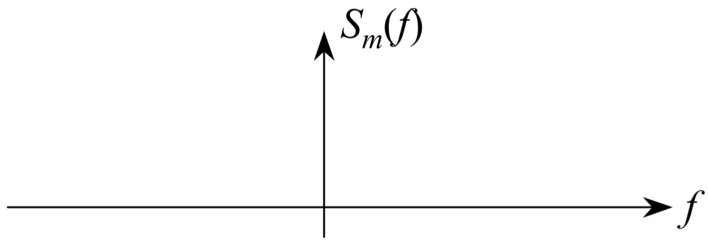






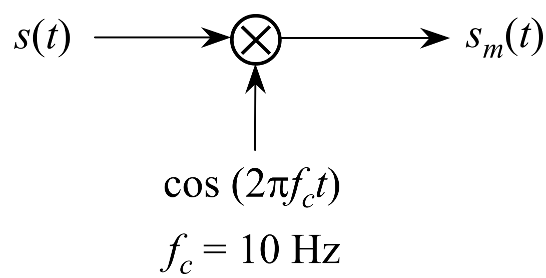
- Transmission constraints ?

- Receiver



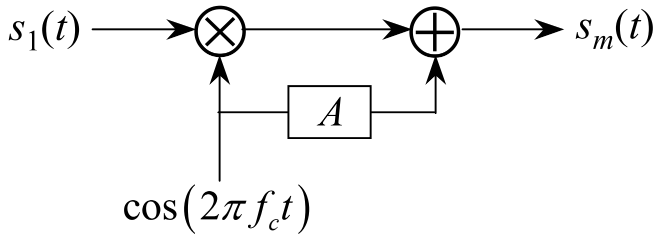
- Example: information signal

$$s(t) = \frac{\sin(2\pi t)}{t}$$



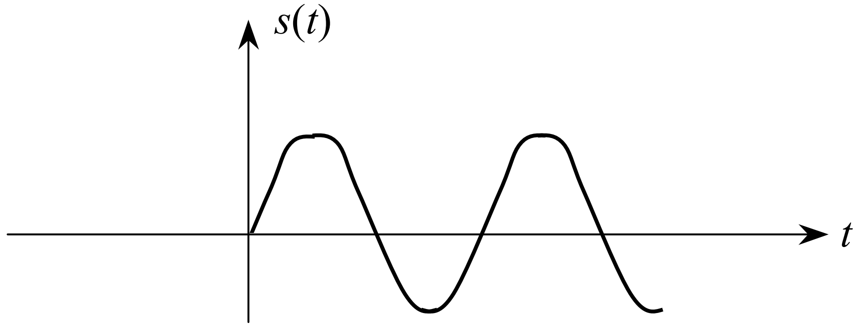
1) Plot $s_m(t)$
2) Compute and plot $S_m(f)$
3) Design the receiver needed to recover $s(t)$ from $s_m(t)$

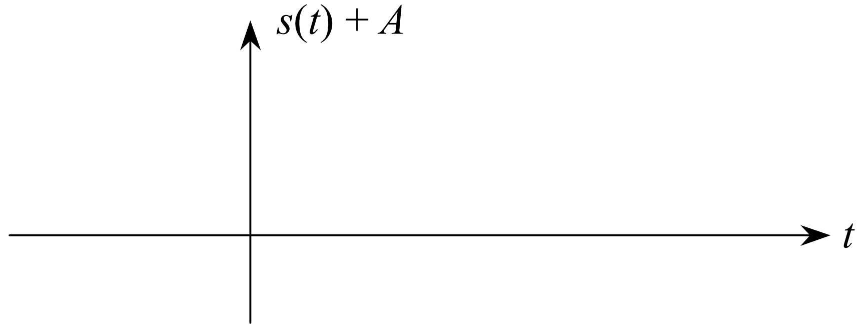
2) Double Sideband Transmitted Carrier

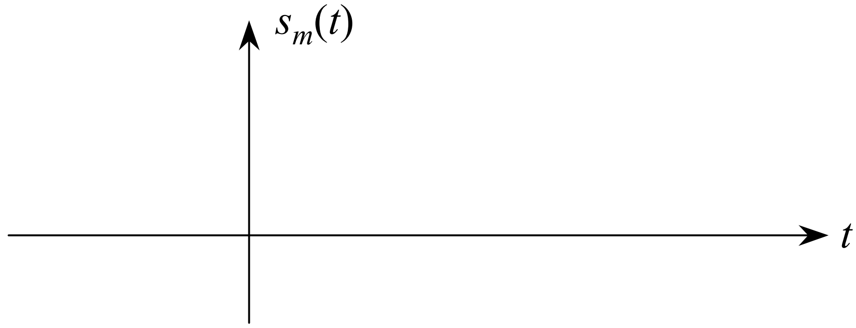


$s_m(t) =$

$S_m(f) =$







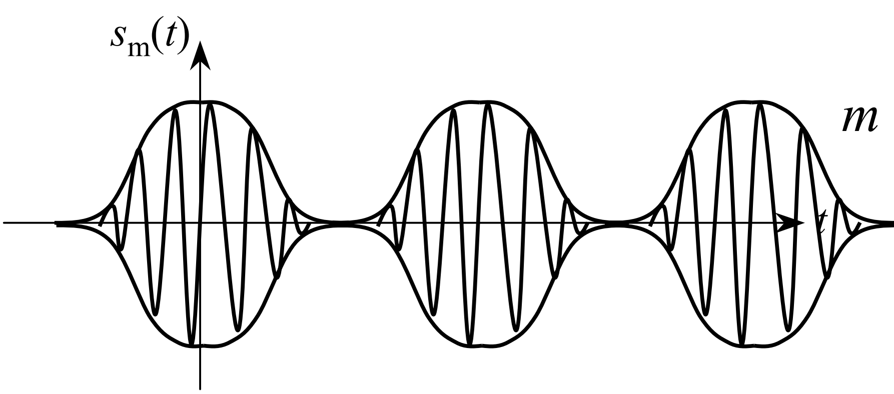
- Modulation Efficiency

efficiency $\eta = \dfrac{\text{signal power}}{\text{total power}}$

$=$

- Modulation Index

$$m = \frac{\max\left|s\left(t\right)\right|}{A}$$



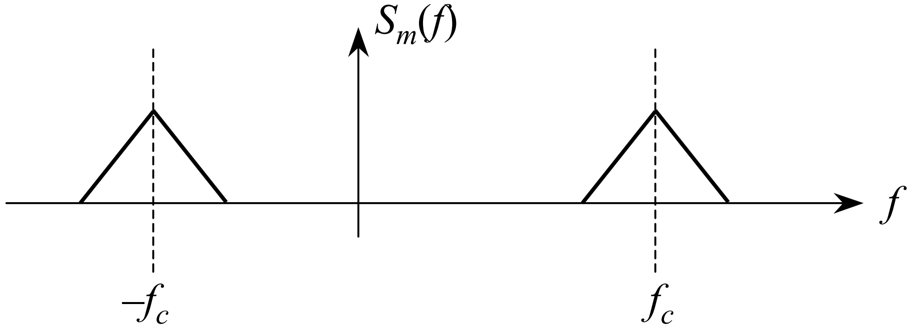
$m =$

$$s\left(t\right) = \cos\left(2\pi f_0 t\right)$$

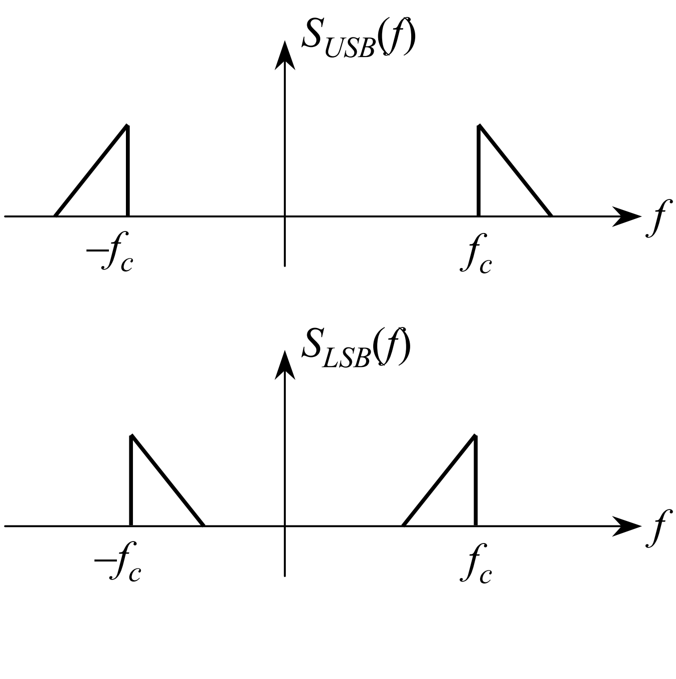
$$A = ?$$

3) Single Sideband

- Recall double sideband AM



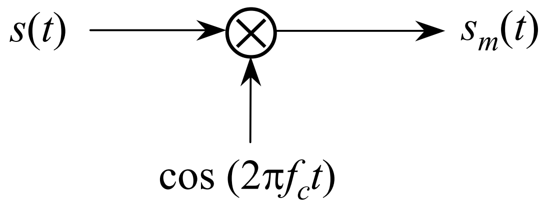
Are both sides really needed ?

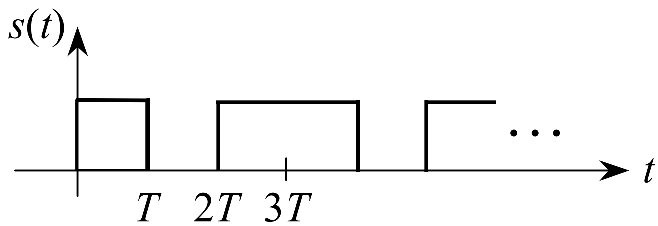


4) Vestigial Sideband (VSB)

- SSB needs less frequency bandwidth than DSB
- SSB transmitter and receivers are complicated (expensive)
- VSB is a trade-off between DSB and SSB

5) Amplitude Shift Keying (ASK)





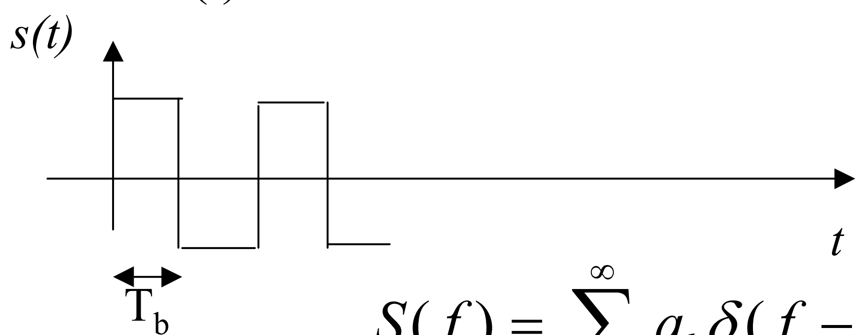
- ASK Spectrum $S_m(f)$

  - Typical s(t) not periodic, due to random “0” & “1” bits

    1) Periodic *s(t)* case

    2) Extension to non periodic *s(t)* case

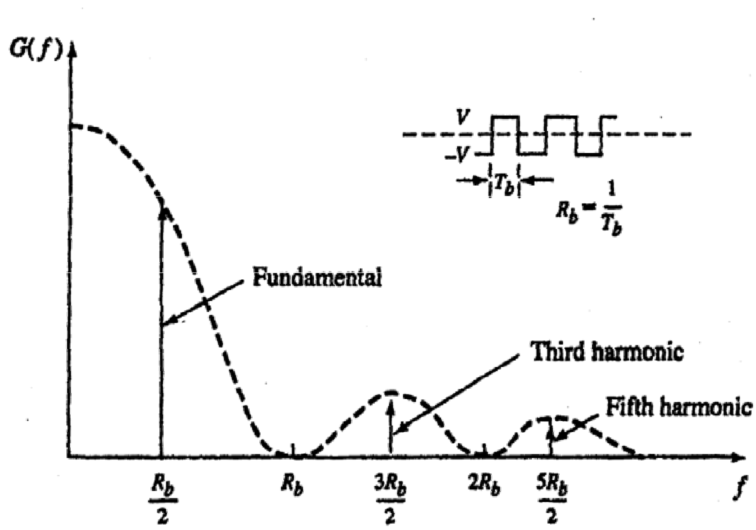
1) Periodic s(t): 1 0 1 0 1



$T_b=T/2$

$$S(f)=\sum_{k=-\infty}^{\infty} a_k \delta(f-kf_0)$$

Recall Delta’s are located at $kf_0=k/2T_b=kR_b/2$

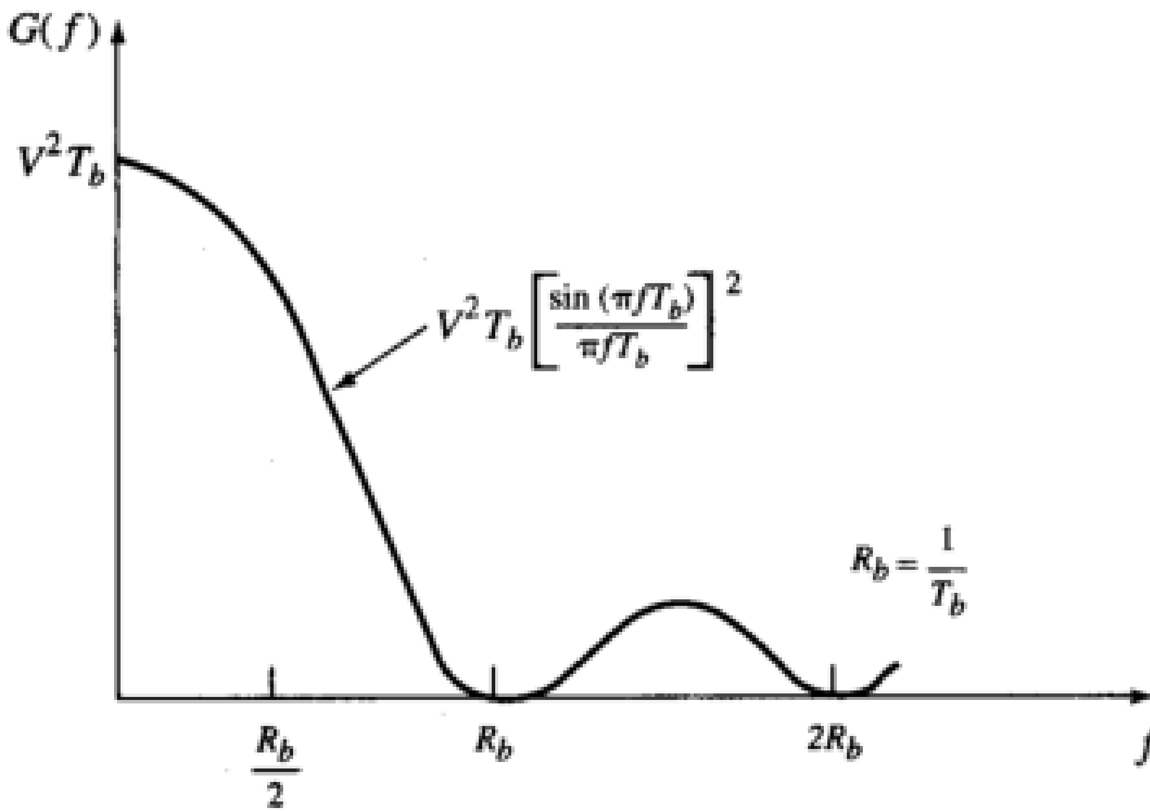


2) Extension to non periodic *s(t)* case

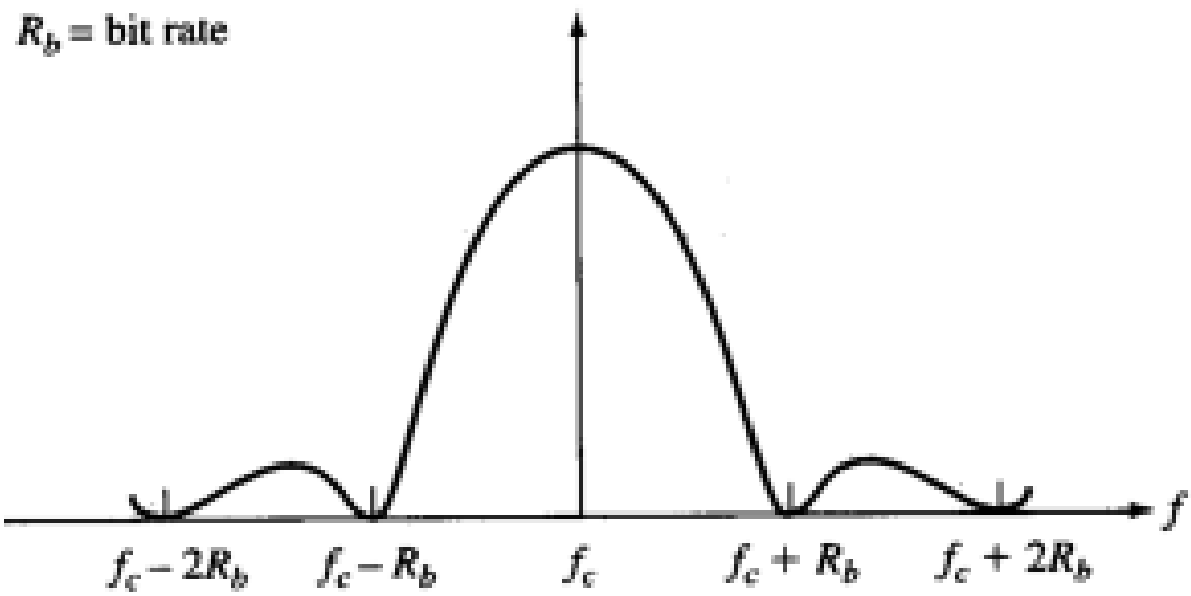
Recall the Power Spectrum of random NRZ was defined as (section II.A):

Def: The power spectral density (PSD) for a non periodic signal s(t) is defined as:

$$G(f) = \lim_{\Delta T \to \infty} \left(\frac{1}{\Delta T} \left| \int_{-\Delta T/2}^{\Delta T/2} s(t) e^{-j2\pi ft} dt \right|^2 \right)$$



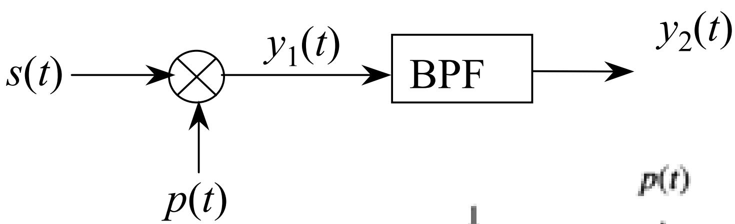
- ASK Spectrum for random bits signal

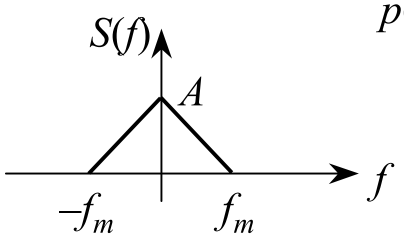


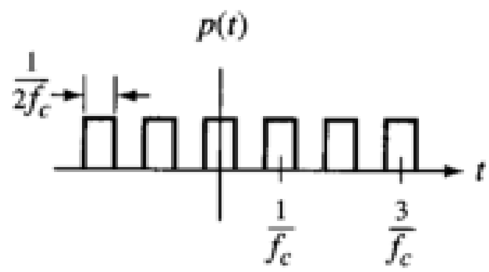
7) Modulators

a) Gated modulator

- Introduction

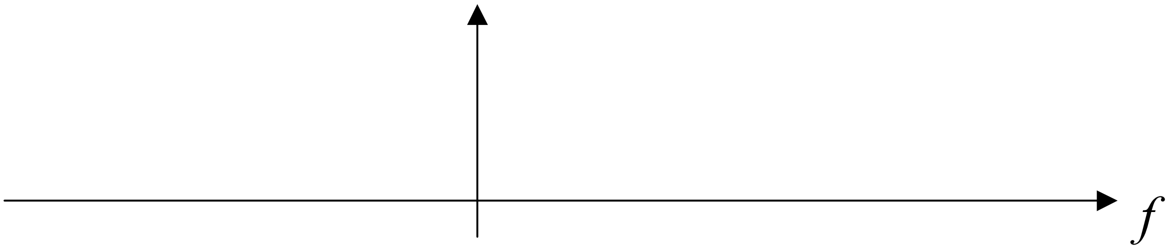






$y_1(t) =$

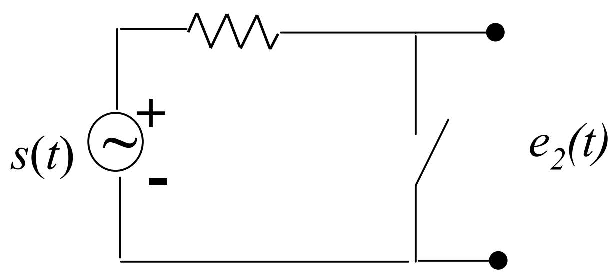
$Y_1(f) =$



- Implementation

Note:

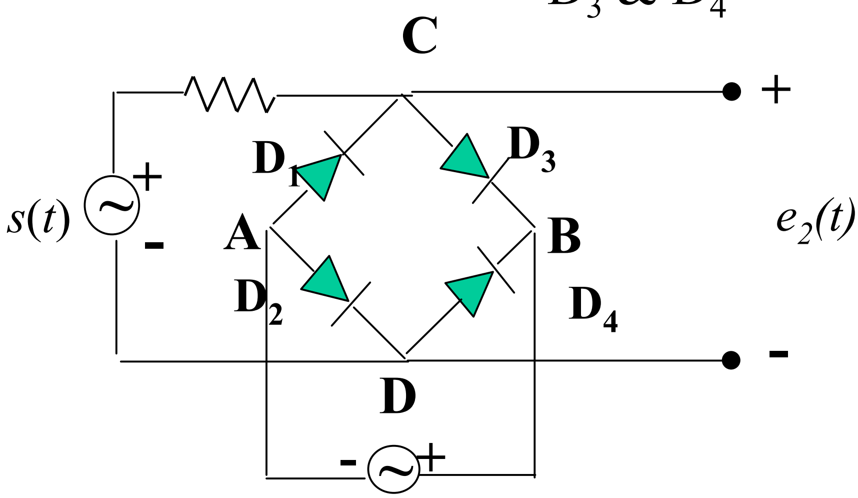
- we need to implement s(t).p(t) with $p(t) = \begin{cases} 0 \\ 1 \end{cases}$



•Process corresponds to a switch operating at a rate of f_c times/sec

•Too fast for a mechanical switch, must be electric.

D_1 & D_2 are a matched pair
D_3 & D_4

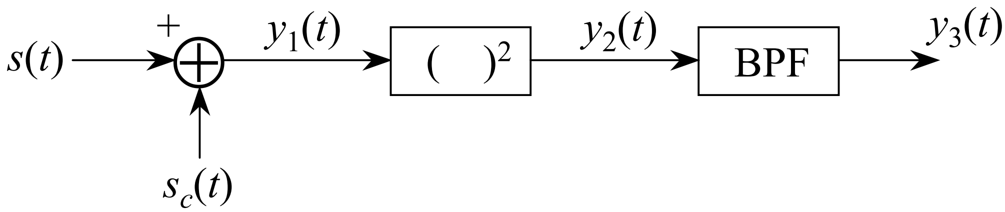


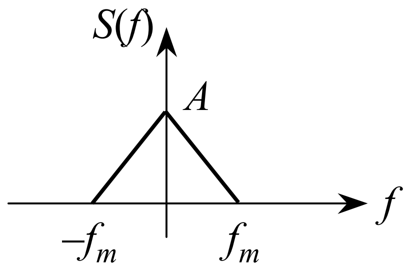
$A\cos(2\pi f_c t)$

1) $A\cos(2\pi f_c t)>0$ =>

1) $A\cos(2\pi f_c t)<0$ =>

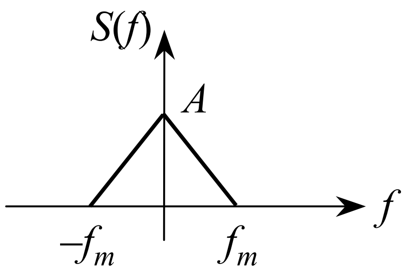
b) Square law modulator

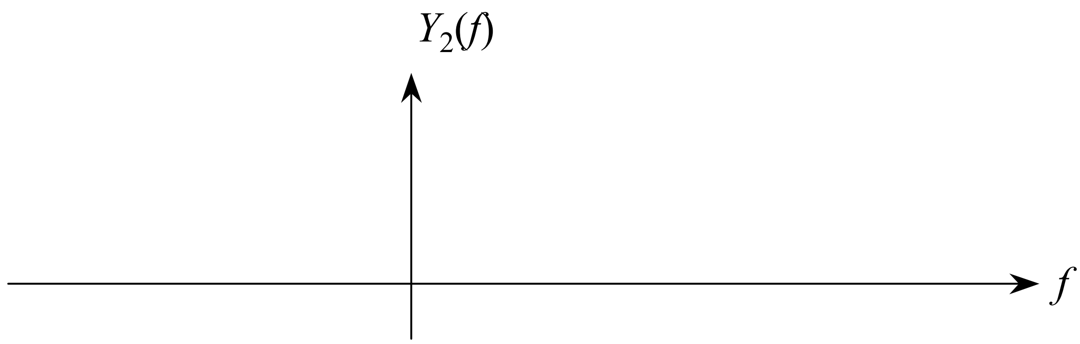


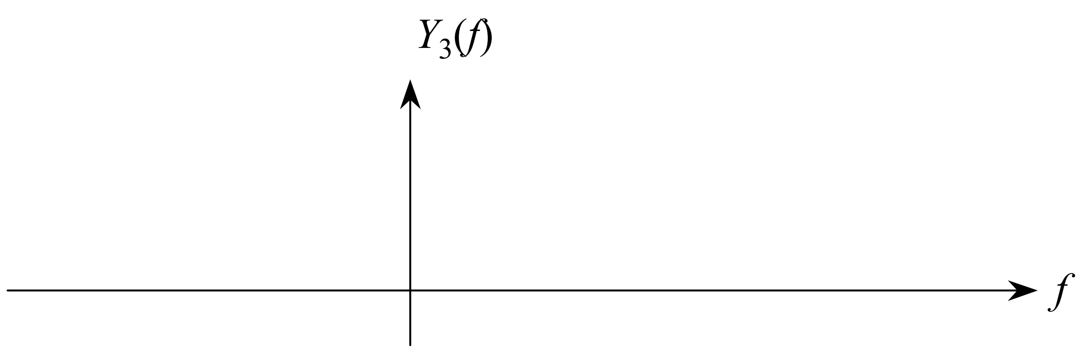


$y_2(t) =$

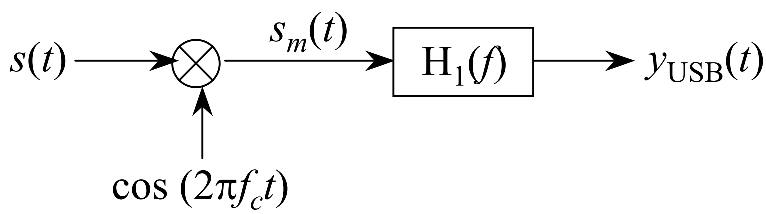
$Y_2(f) =$

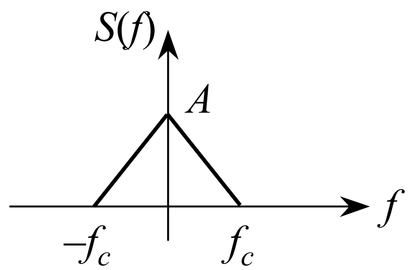

S(f)
A
f
−fm
fm

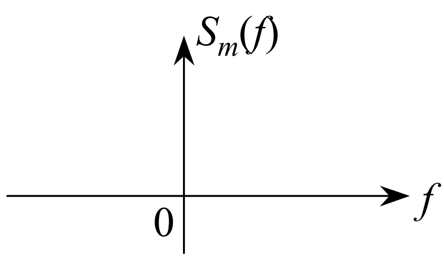

Y2(f)
f

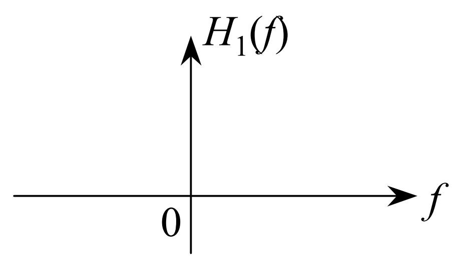

Y3(f)
f

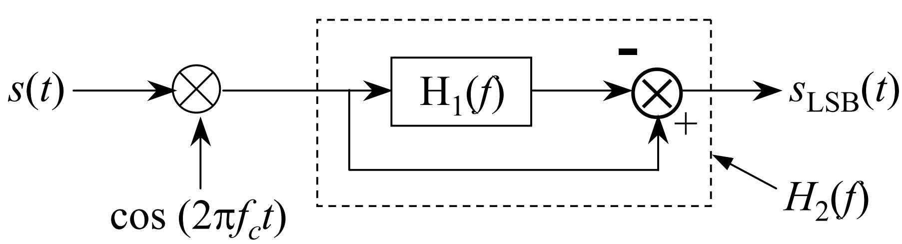
b) SSB modulator

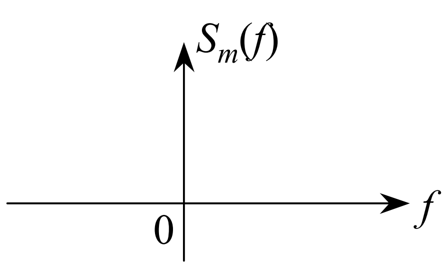


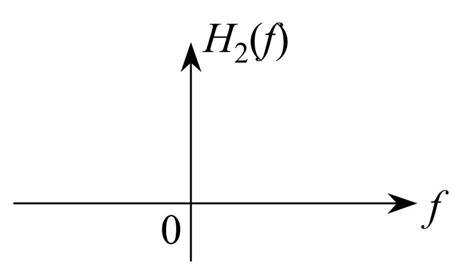






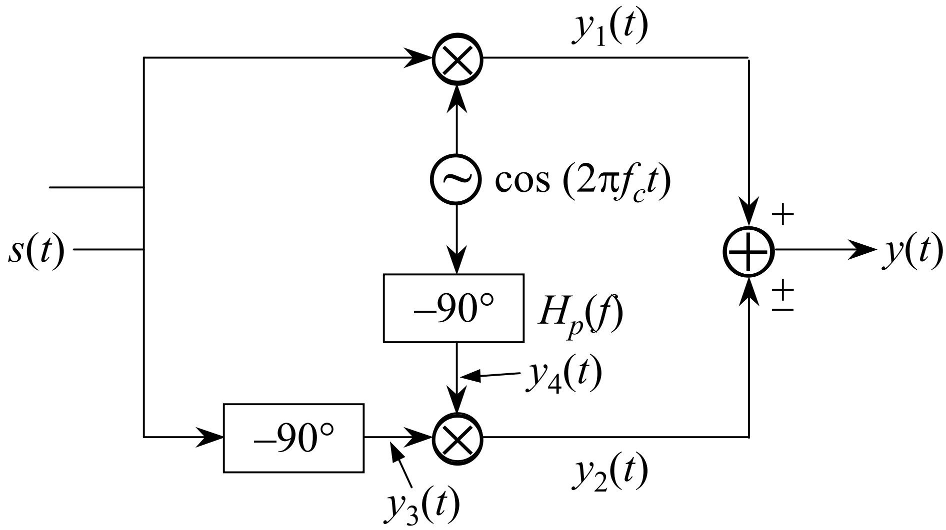


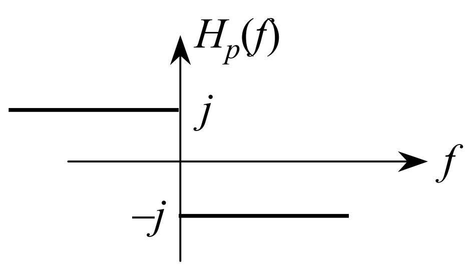


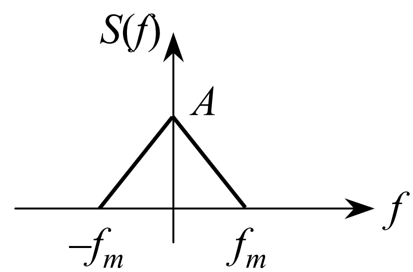


- Potential problem with above SSB modulator

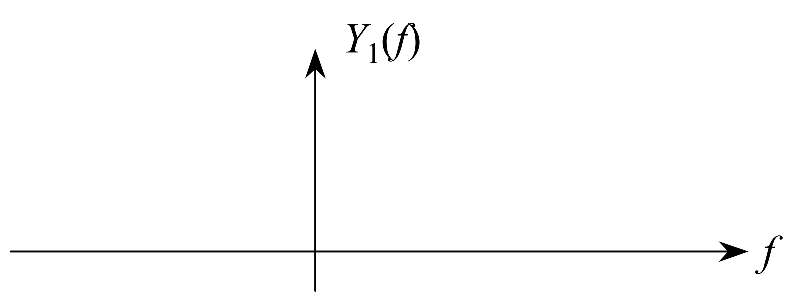
- Phase shift SSB modulator

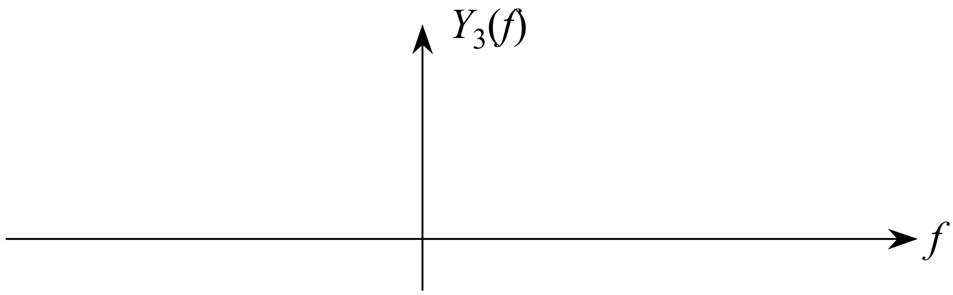


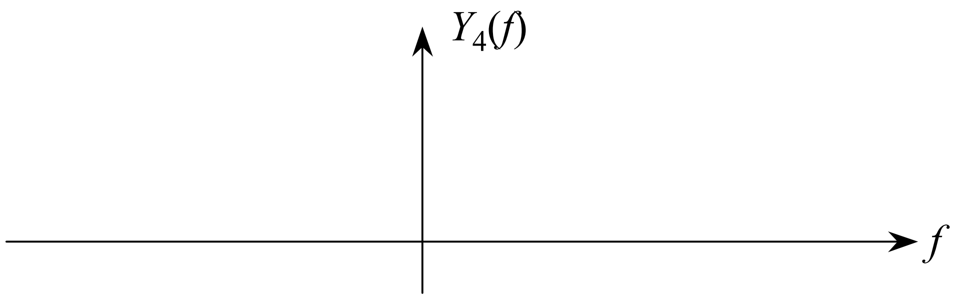


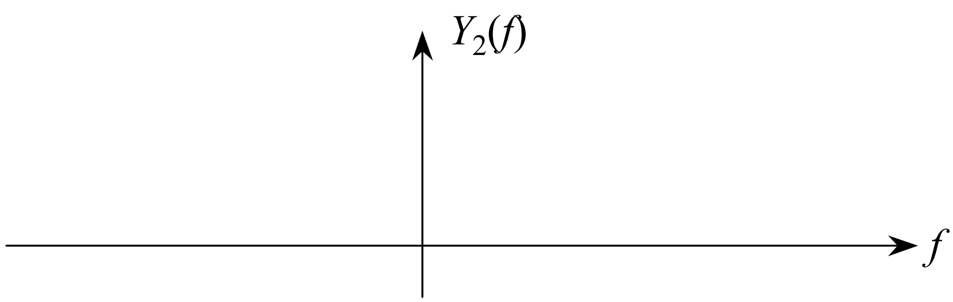


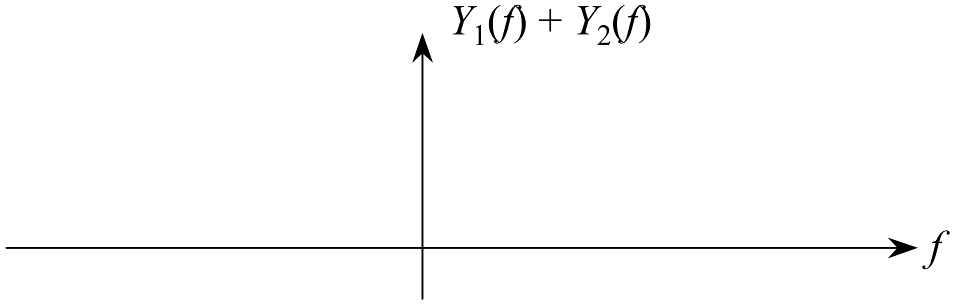
$$H_p(f) = -j \operatorname{sgn}(f)$$

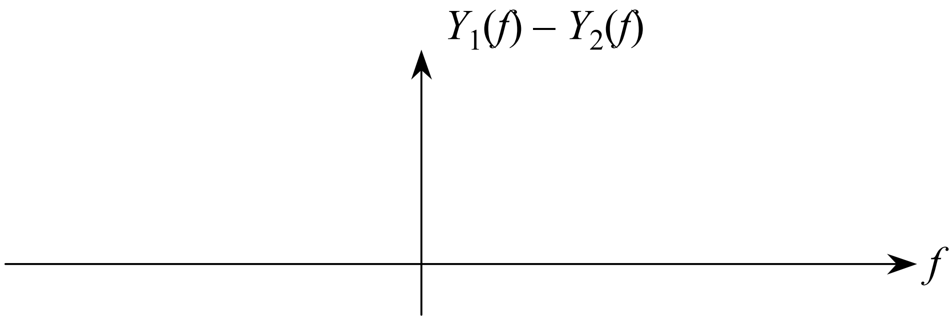



$Y_3(f)$
f

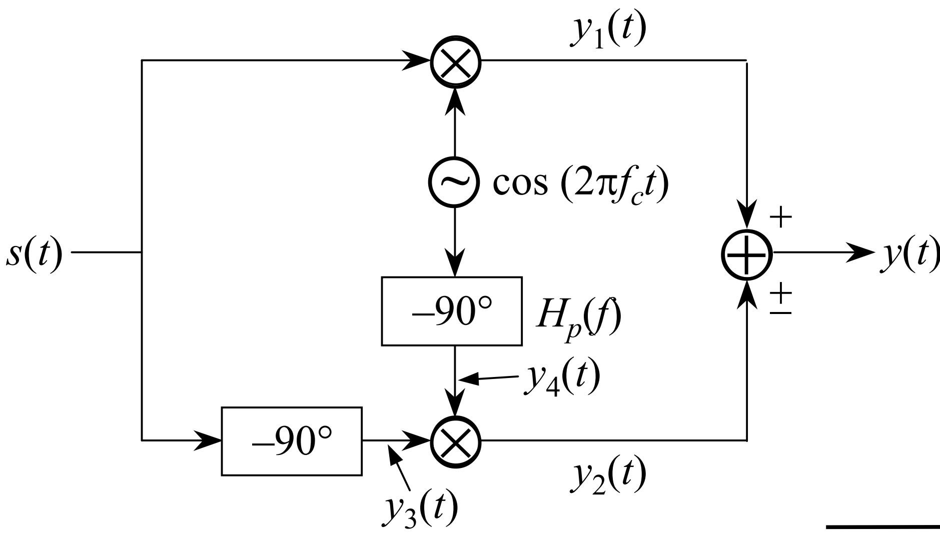

$Y_4(f)$
f

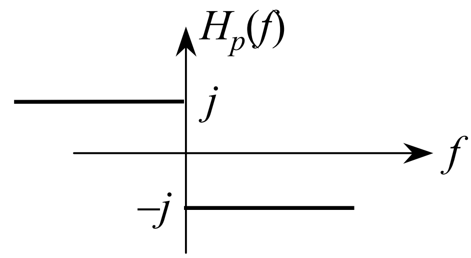

$Y_2(f)$
f


$Y_1(f) + Y_2(f)$
f


$Y_1(f) - Y_2(f)$
f

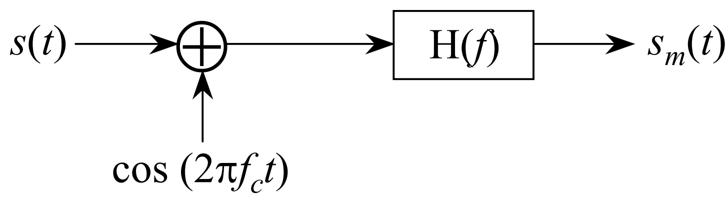
• SSB time domain expression





• SSB frequency domain expression

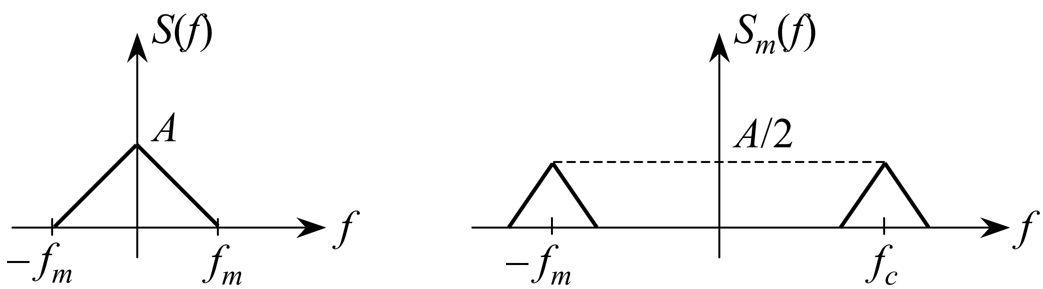
- Vestigial Sideband (VSB) modulator

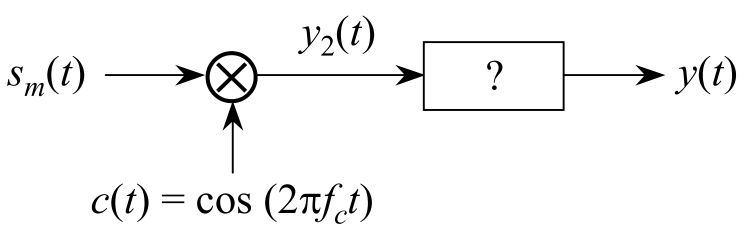


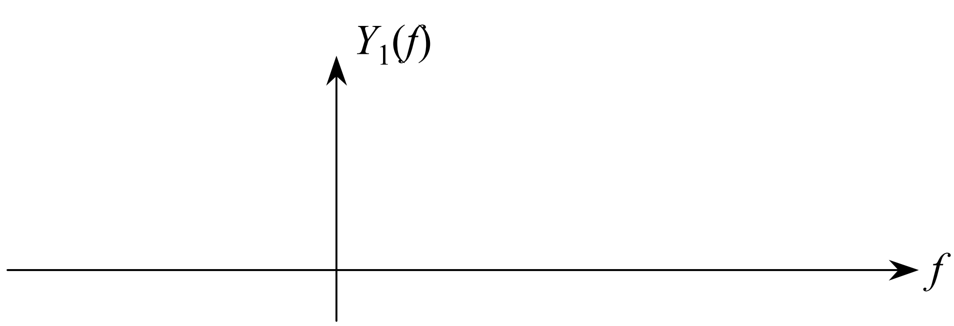
8) Demodulators

- Two different types:
  - coherent: requires synchronization
  - incoherent: simple to implement

a) Coherent demodulation





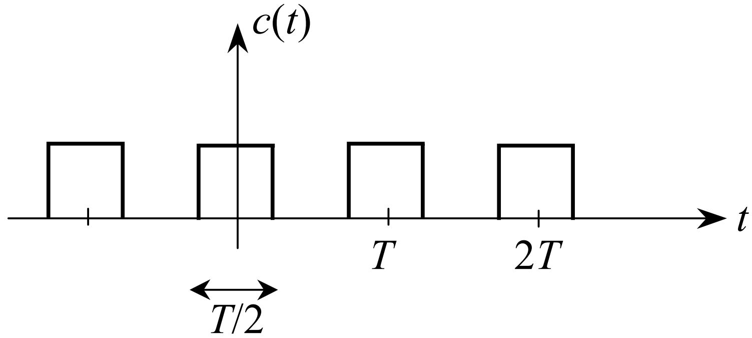


$$y_2(t) =$$

✶ Gated Demodulator

- Do we have to use a cos $(2\pi f_c t)$ to recover $s(t)$?

  Assume $c(t)$ defined as



✶ Square Law Demodulator

$$s_m(t) \longrightarrow \boxed{(\,.\,)^2} \longrightarrow y(t)$$

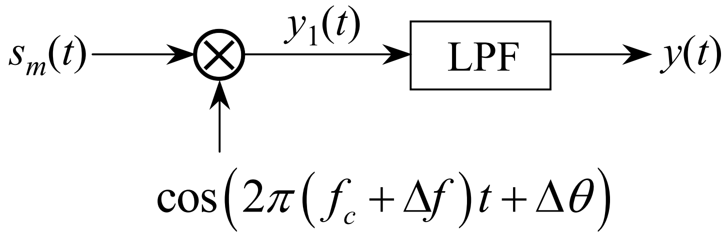
$$y(t) = \left(s_m(t) + A\cos(2\pi f_c t)\right)^2$$

DBB-WC modulation case

$s_m(t)=$

$y(t)=$

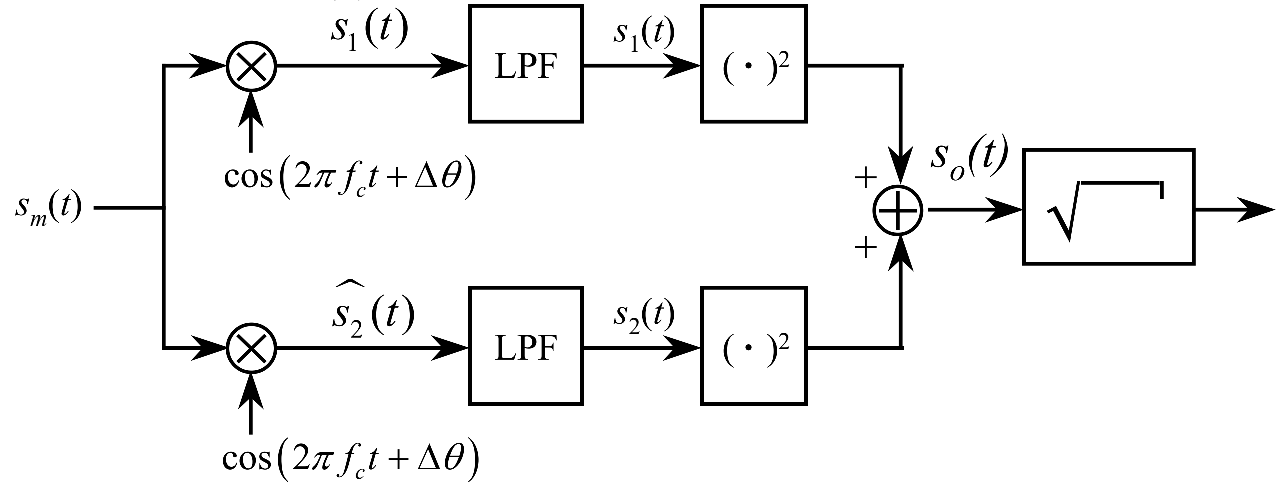
✶ Frequency & Phase Mismatch Effects



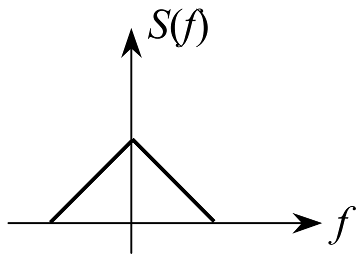
$$y_1(t) = \left(s(t)\cos 2\pi f_c t\right)\cos\left(2\pi\left(f_c+\Delta f\right)t+\Delta\theta\right)$$

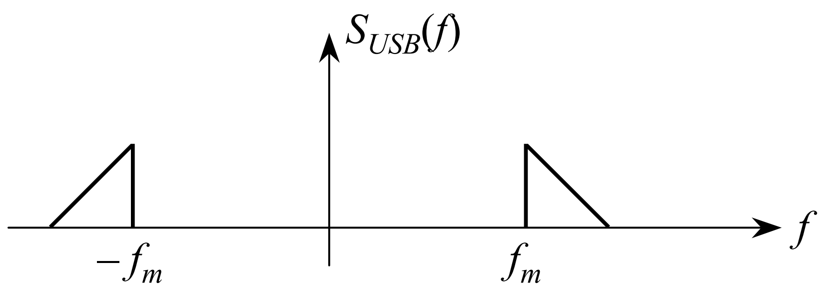
$$=$$

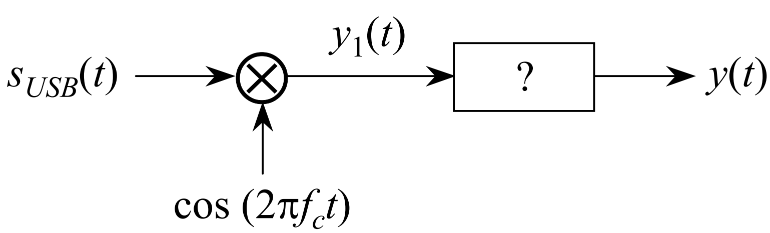
- Quadrature Receiver

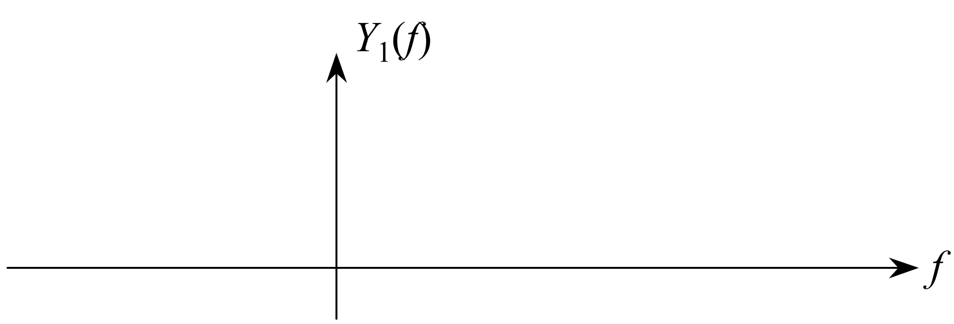


✶ Single Sideband Demodulation







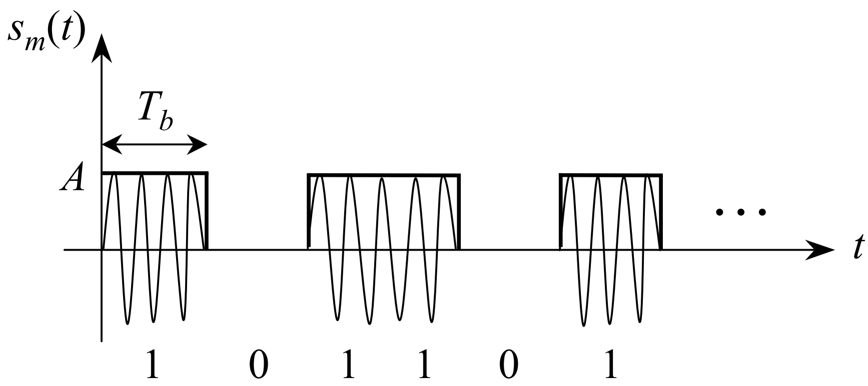


$$y_1(t) =$$

✶ Vestigal Sideband Demodulation

$$S_V\left(f\right) = S_m\left(f\right) H\left(f\right) =$$

- ASK Demodulation



- We know which signal shape is sent originally

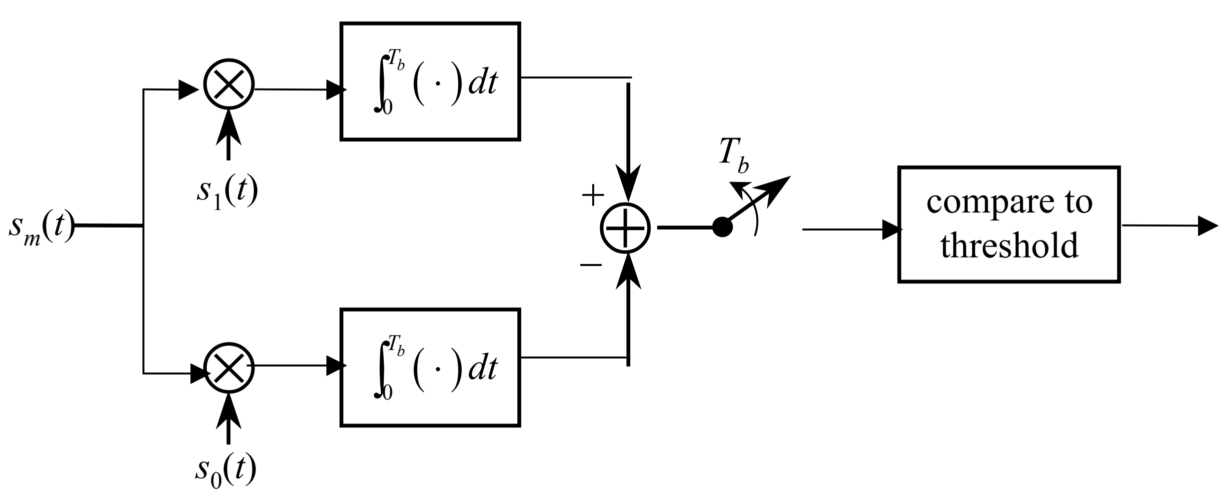
↓

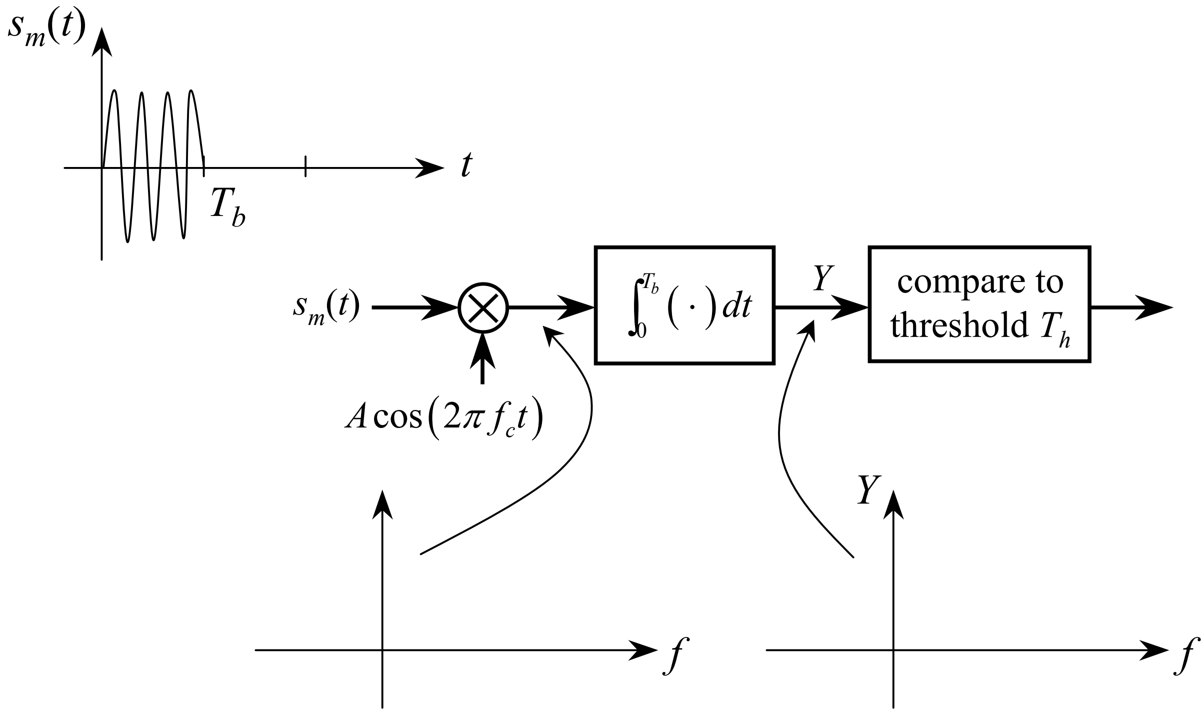
take advantage of it

↓

- Matched filter detector

Recall: binary matched filter detector





- How to compute the threshold T_h ?

$$Y = \int_0^{T_b} A s_m(t)\cos(2\pi f_c t)\,dt \qquad \text{when } s_m(t) = A\cos 2\pi f_c t$$

$$= \int_0^{T_b} A^2 \cos^2(2\pi f_c t)\,dt$$

$$= A^2 \int_0^{T_b}\left[\frac{1+\cos(4\pi f_c t)}{2}\right]dt$$

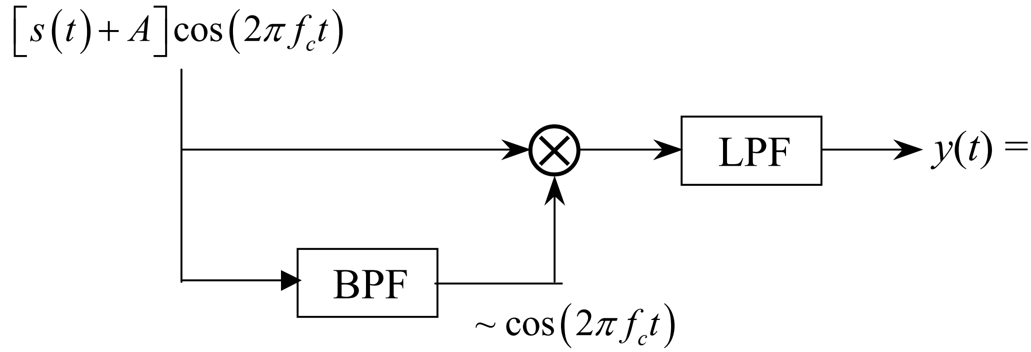
$$Y = \begin{cases} \dfrac{A^2 T_b}{2} & \text{when } s_m(t) = A\cos 2\pi f_c t \\ 0 & \text{when } s_m(t) = 0 \end{cases}$$

Threshold = ________________

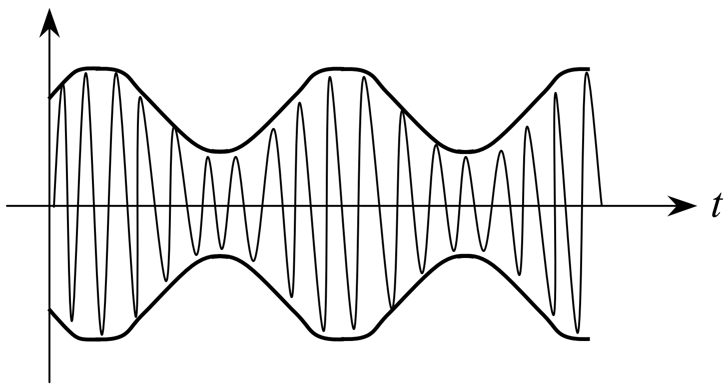
- Carrier frequency recovery in AMTC

  when f_c is transmitted

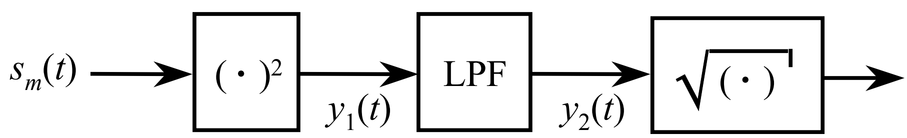
  - narrowband BP filter
  - phase lock loop



b) Incoherent (asynchronous) Demodulation



- Square Law Detector



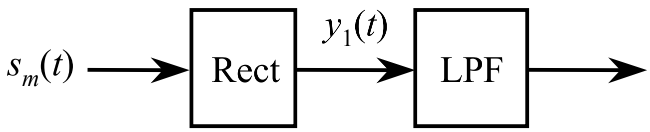
$y_1\left(t\right) =$

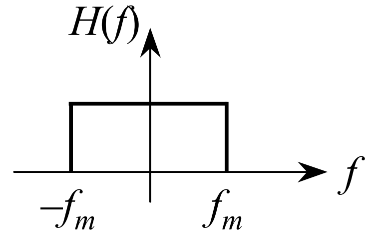
$y_2\left(t\right) =$

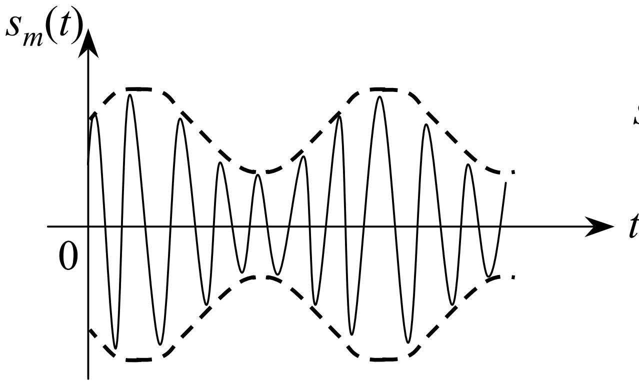
- LPF cutoff frequency: __________

- Constraint needed on signal amplitude:

- Rectifier Detector





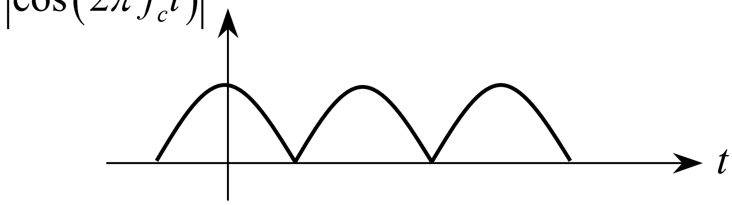


$$s_m(t) = \left(A + s(t)\right)\cos\left(2\pi f_c t\right)$$

$$y_1(t) =$$

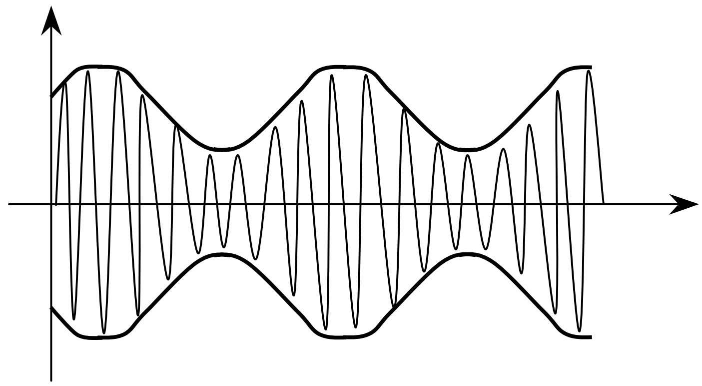
- Expand $\left|\cos\left(2\pi f_c t\right)\right|$ in a Fourier series expansion

Fundamental period for $\left|\cos\left(2\pi f_c t\right)\right|$

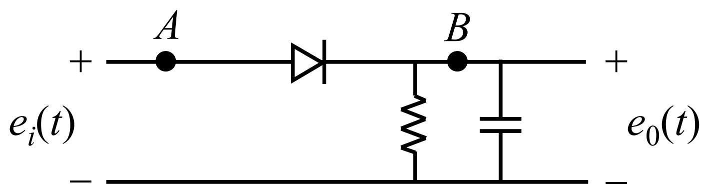


$$\left|\cos\left(2\pi f_c t\right)\right| =$$

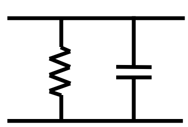
- Envelope Detector

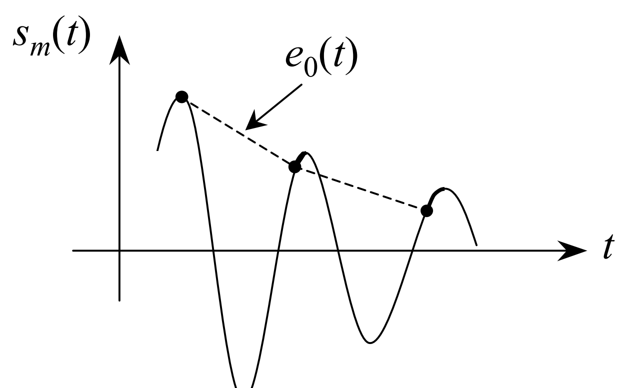


Need to follow envelope only !



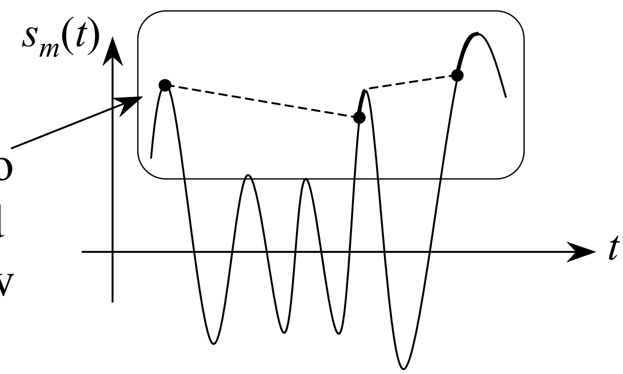
(1) $e_{\mathrm{i}}(t) < e_0(t)$ $V_A < V_B$ diode shuts off capacitor discharges in R	(2) $e_{\mathrm{i}}(t) \geq e_0(t)$ $V_A \geq V_B$ diode transmits capacitor loads





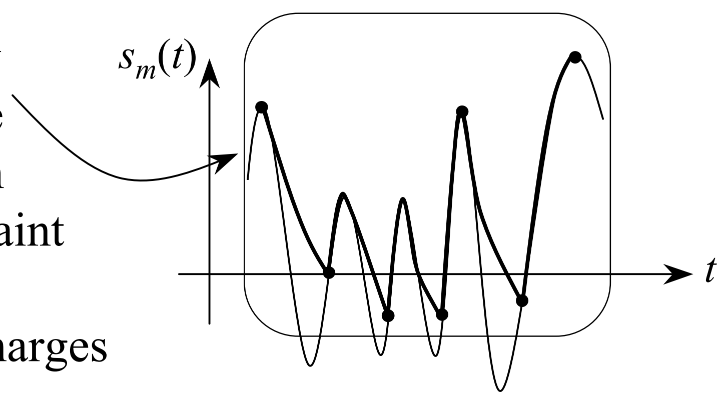
- Envelope Detector Constraints
  - filter RC detector time constraint must be small enough to be able to track changes in $s_m(t)$ peak values

Problem when RC time constraint is too large! Demodulated signal doesn't follow envelope.



  - filter RC detector time constraint must be large enough to follow the envelope trend only.

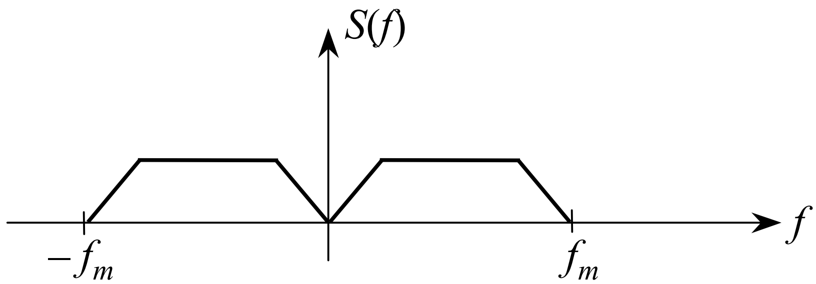
High frequency components are generated when RC time constraint is too small (capacitor discharges too fast).

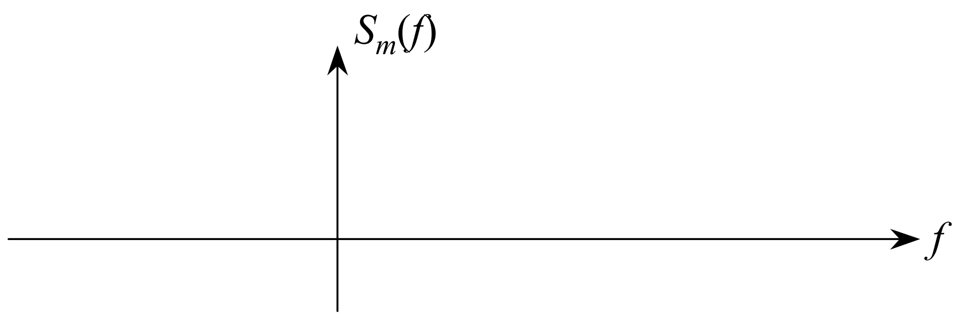


What is the maximum charge in prior values in $s_m(t)$?

- Single Sideband (non-coherent) Demodulator

  (1) Add carrier to make demodulation easier.





$$\rightarrow \quad s_{LSB}(t) + A\cos(2\pi f_c t)$$

$$= \frac{s(t)\cos 2\pi f_c t + \hat{s}(t)\sin 2\pi f_c t}{2} + A\cos(2\pi f_c t)$$

$$= \left[\frac{s(t)}{2} + A\right]\cos 2\pi f_c t + \frac{1}{2}\hat{s}(t)\sin(2\pi f_c t)$$

(2) Use envelope detector to recover information signal.

→ envelope of

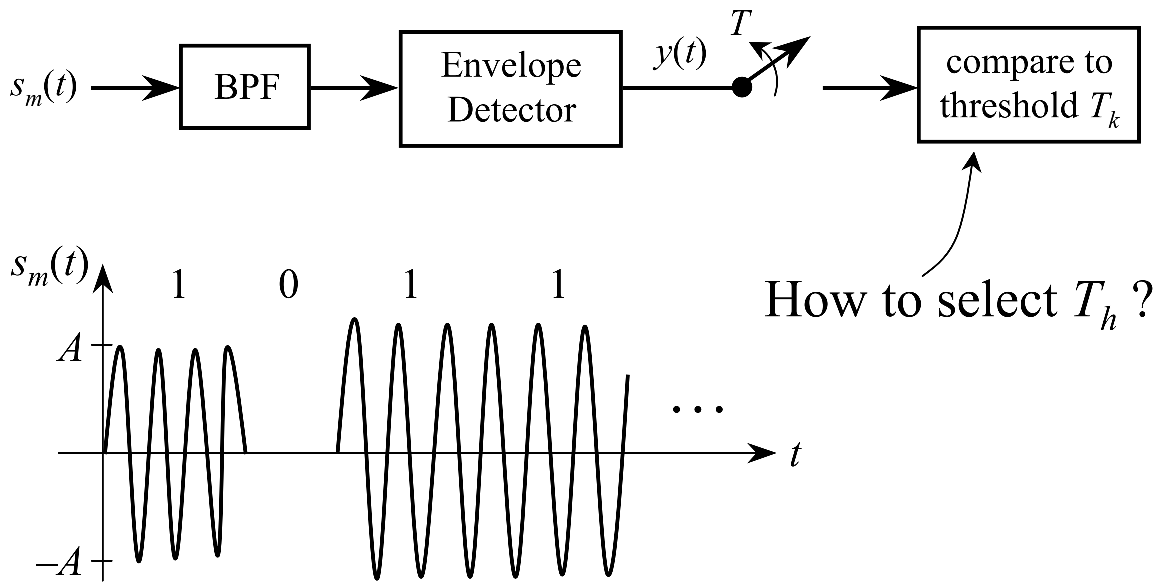
$$\left[\frac{s(t)}{2}+A\right]\cos 2\pi f_c t+\frac{1}{2}\hat{s}(t)\sin(2\pi f_c t)$$

⇓

$$E=\sqrt{\left(\frac{s(t)}{2}+A\right)^2+\left(\frac{1}{2}\hat{s}(t)\right)^2}$$

$$\simeq \frac{s(t)}{2}+A \qquad \text{when} \quad A\gg s(t)$$

- ASK Incoherent Demodulator



9) Applications to the AM superheterodyne receiver

- Application of modulation property

$$p(t) = s(t) \cdot \cos 2\pi f_0 t$$

$$\swarrow$$

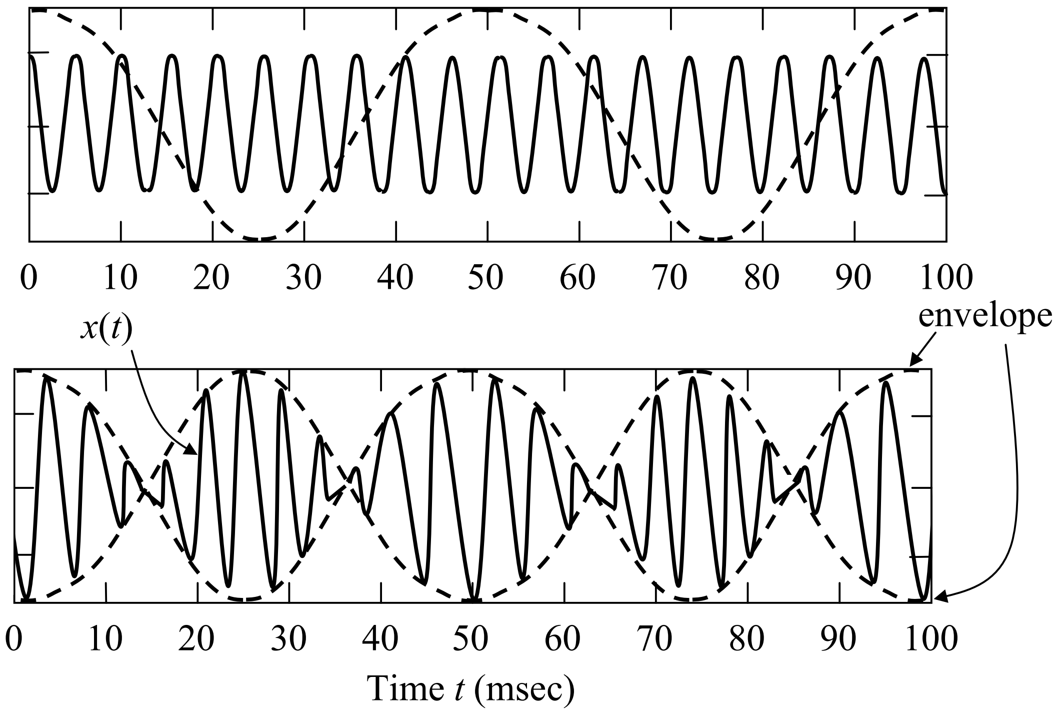
$$P(f) =$$

- Applications: communication systems; Amplitude modulation (AM) systems.
- Speech exist in the range 300Hz~5KHz
- Atmosphere attenuates signals rapidly in the range 10Hz-->20KHz, and propagates much better at high frequencies

⇒ shift speech to higher frequency range

• Recall what the AM signal looks like

Example: $x(t) = \cos(40\pi t) \cdot \cos(400\pi t)$

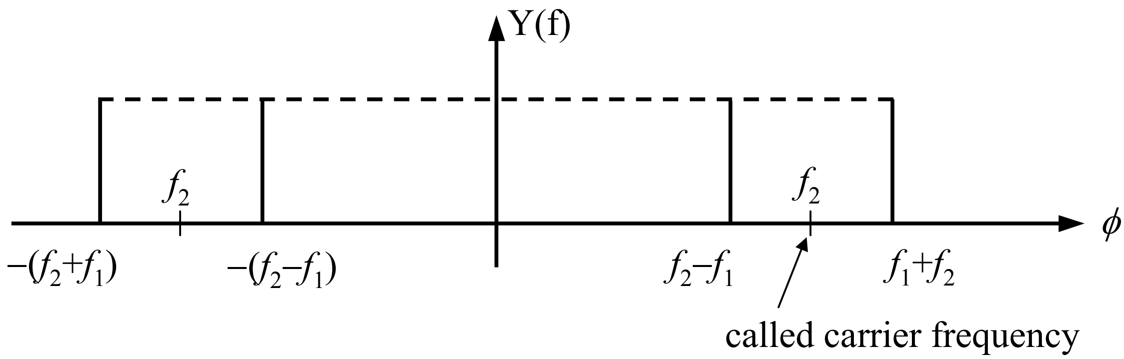


$$x(t) = \frac{1}{2}\left[\cos(440\pi t) + \cos(360\pi t)\right]$$

$$= \frac{1}{4}\left[e^{j440\pi t} + e^{-j440\pi t} + e^{-j360\pi t} + e^{-j360\pi t}\right]$$

Spectrum of $x(t)$ for generic frequencies:

$$y(t) = \cos(2\pi f_1 t)\cos(2\pi f_2 t) \qquad f_2 \gg f_1$$
$$= \frac{1}{2}\left[\cos 2\pi (f_1 + f_2)t + \cos 2\pi (f_1 - f_2)t\right]$$
$$= \frac{1}{4}\{\exp\left[j2\pi (f_1 + f_2)t\right] + \exp\left[-j2\pi (f_1 + f_2)t\right]$$
$$+ \exp\left[j2\pi (f_1 - f_2)t\right] + \exp\left[-j2\pi (f_1 - f_2)t\right]$$



Note:

Change $f_2 \rightarrow$ you change where the frequency's components are for a constant f_1

- How to recover the original speech signal ?

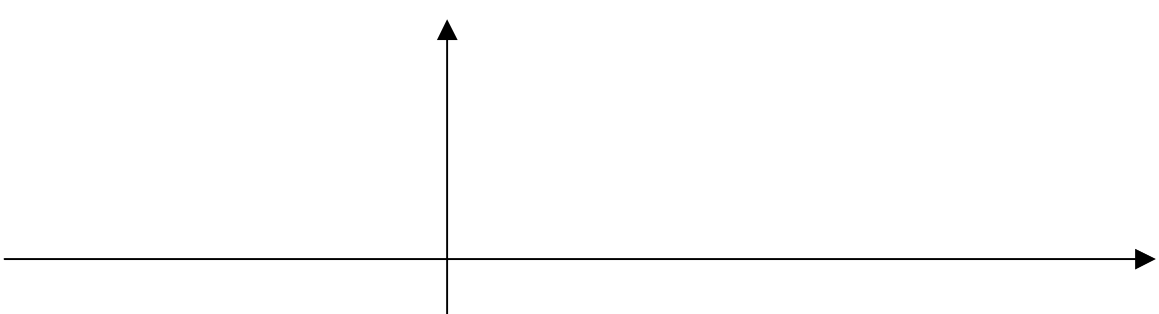
  Demodulate….

Several basic operations are needed in a broadcast receiver:

1. Station separation: must be able to pick out a specific signal and reject others
2. Amplification: needed when the signal picked up by the radio antenna is too weak to drive the loudspeakers
3. Demodulation: The received signal is centered around the carrier frequency and must be demodulated before it is fed into the speakers

In standard AM:

- The maximum audio signal frequency is around 5kHz
- Each station is assigned 10kHz by the FCC (i.e., each adjacent carriers are separated by 10kHz)
- The AM frequency band assignment is 540kHz --> 1600kHz

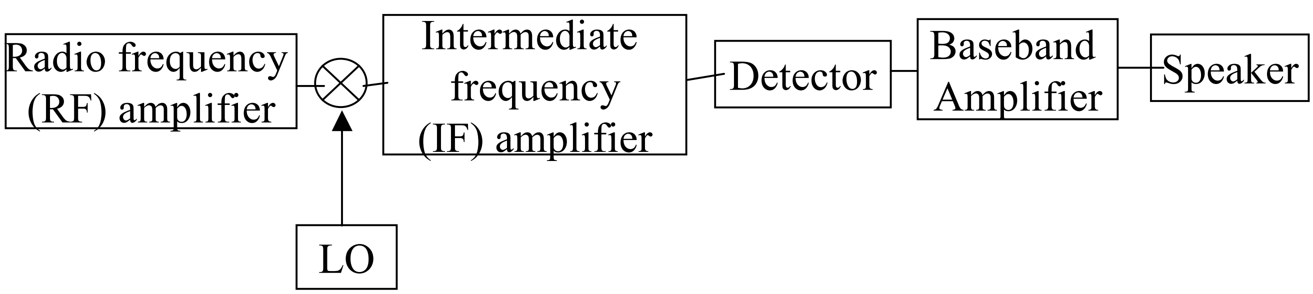


• **Filter constraints:** We need tuneable filters with sharp cutoff frequencies to select the station we want → impossible to realize!

• **What is done instead:** We build a fixed bandpass filter and shift the input frequencies so that the frequencies of interest falls within the fixed passband of the filter

- Such a shifting process is called *heterodyning*
- The receiver doing this operation is called a *superheterodyne receiver*

• **Basic AM superheterodyne receiver diagram**



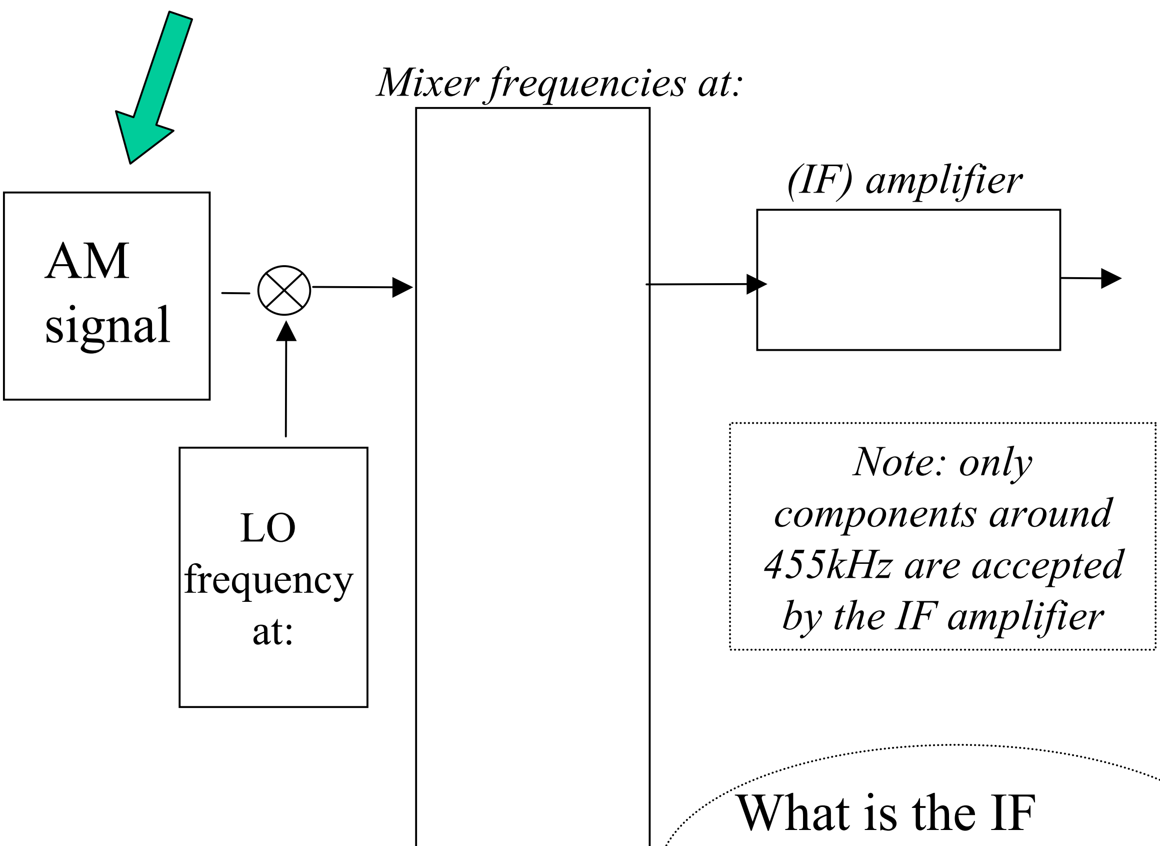
Receiver Components:

- RF amplifier: amplifies a portion of the spectrum (tuneable)
- Local oscillator (Mixer): shifts the signal to a specific frequency range
- IF amplifier: filters and amplifies around a fixed freqency (for AM systems around 455kHz)
- Detector: demodulates (i.e., extracts) the audio signal
- Baseband amplifier: amplifies the audio signal

Example:

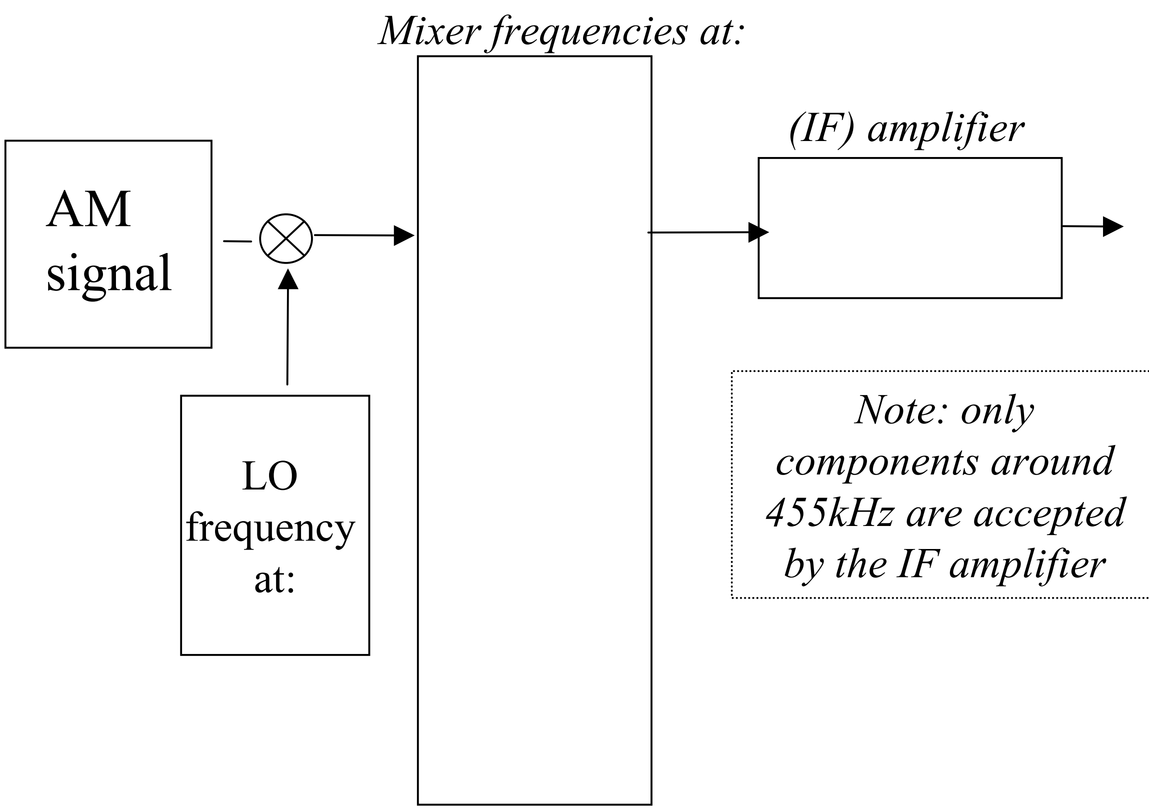
- Assume no RF amplifier (will be added and discussed later)
- Consider the case of a 1kHz AM wave modulated by a carrier at 1MHz (i.e., the station center frequency is at 1MHz)

The generated AM signal has frequencies at:



What is the IF amplifier required bandwidth? ________

• What happens if we want to accept a station located at 1600kHz ? Assume the IF amplifier is centered around 455KHz.



Summary:

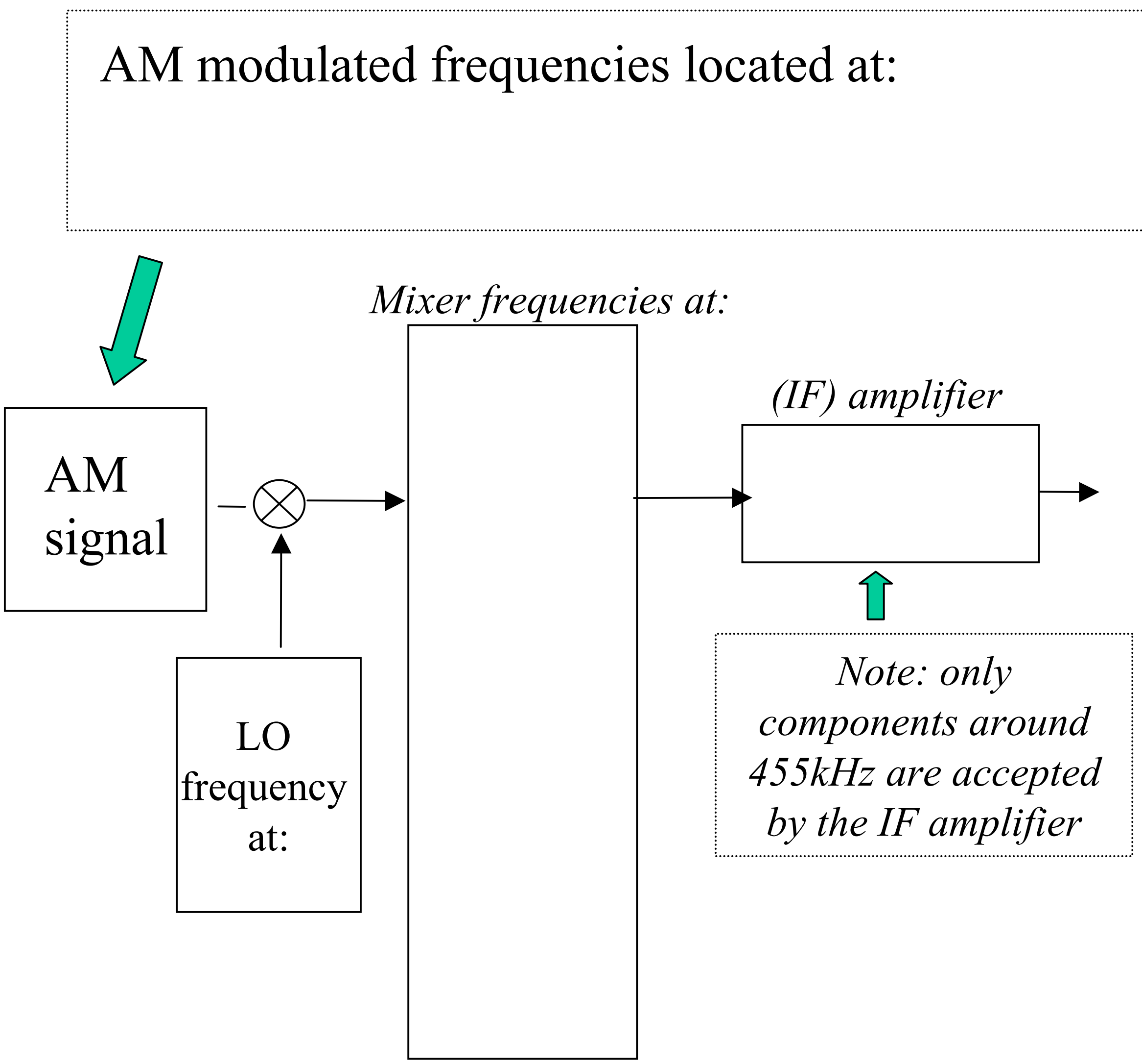
- The key is to make the LO track the incoming signal so hat the difference between the incoming signal and the LO frequency is a constant frequency (called the IF frequency) equal to 455KHz.

- By convention the LO has to be at a frequency 455KHz above the incoming carrier frequency.

- Once we have the IF amplifier output, we can demodulate.

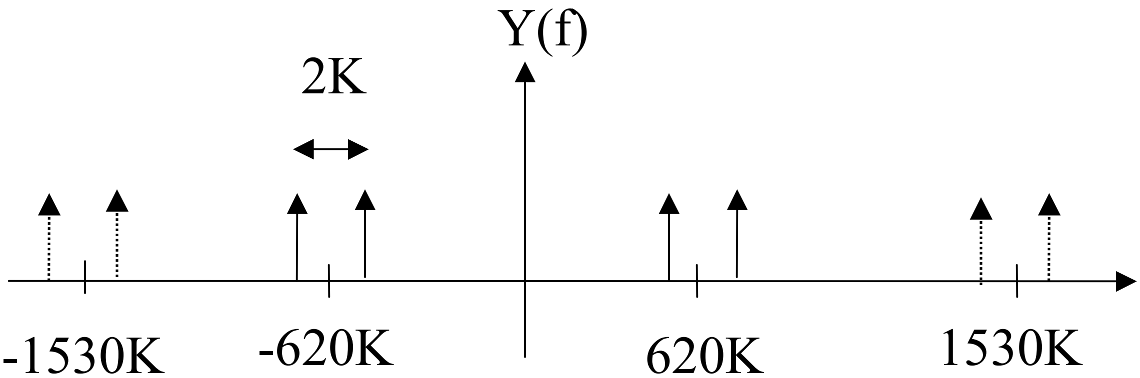
•**The Potential *Image Frequency* problem:** Sometimes we can get a signal other than that desired at the IF amplifier

• Example: Assume we have a desired signal of frequency 1KHz modulated at 620KHz and an undesired signal of frequency 1 KHz modulated at 1530KHz

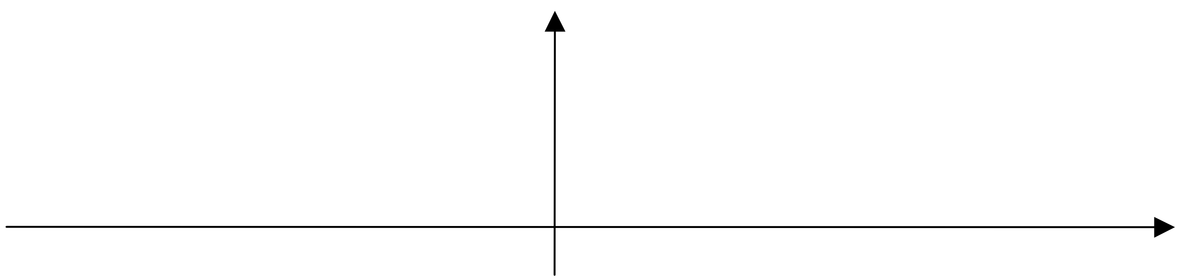


Desired signal modulated @ 620K

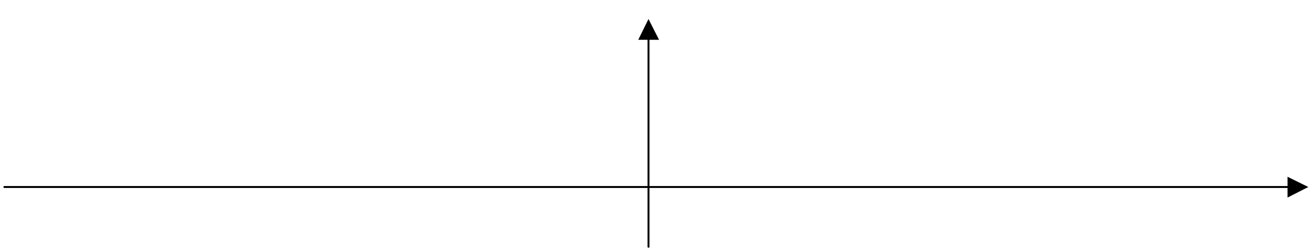
Undesired signal modulated @ 1530K



Shift to the right



Shift to the left

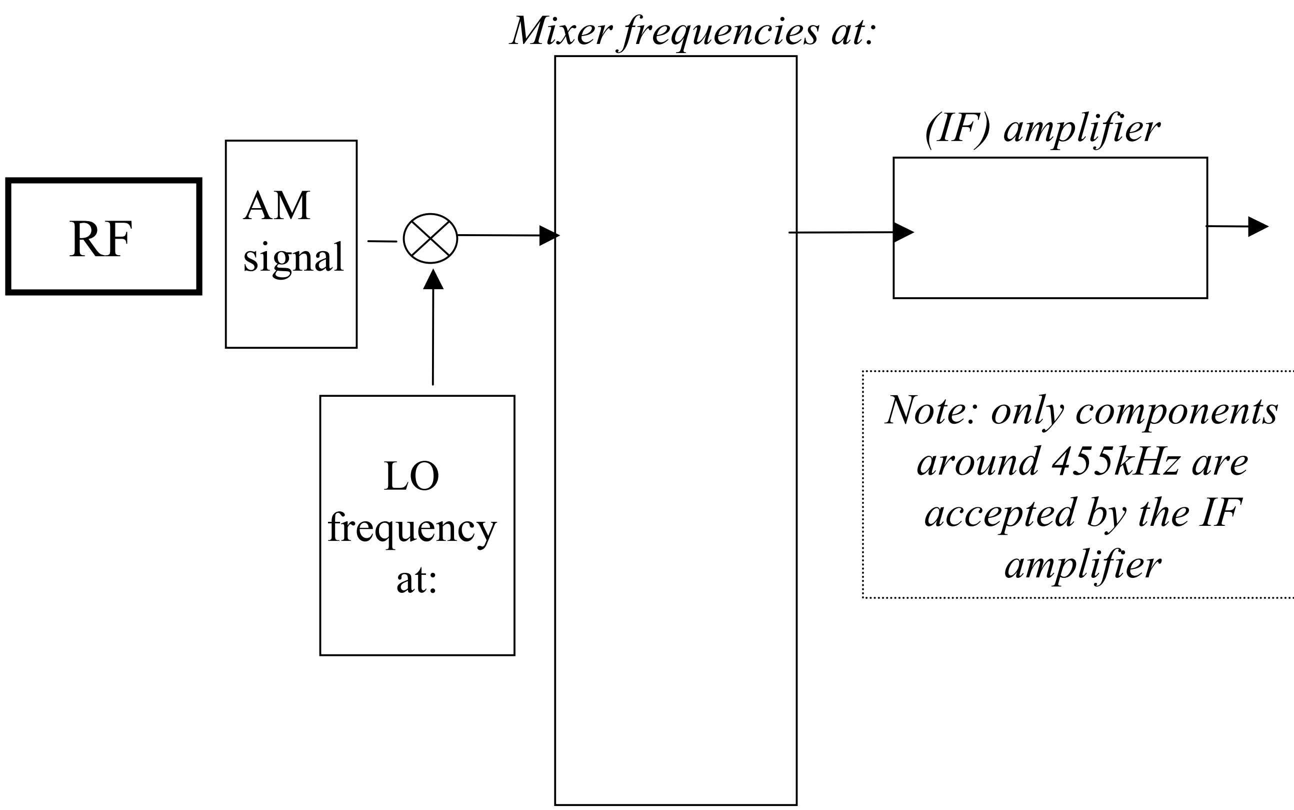


• **Notes**: 1. Both desired and undesired modulated signals have identical components after the mixer. These components won't be separated.

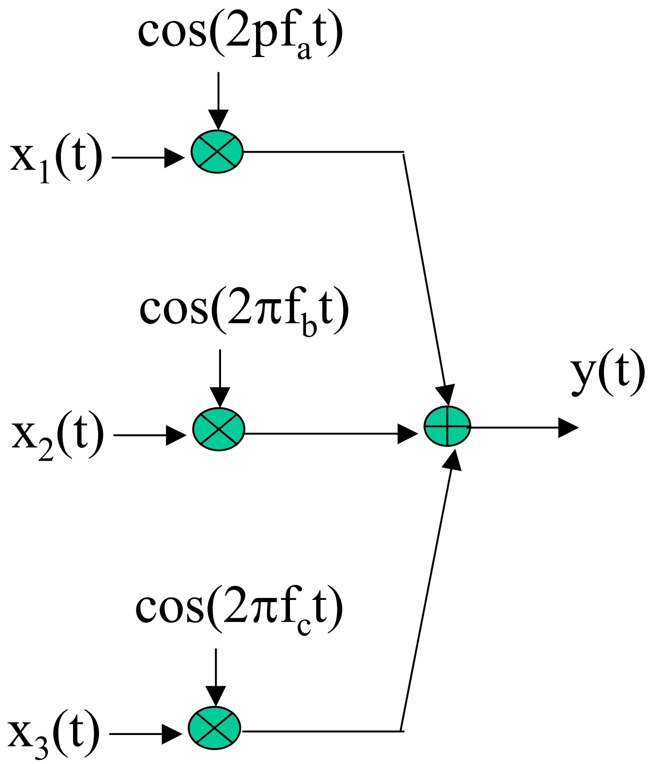
2. Such undesired modulated components are called *image frequencies* (they are the frequencies which appear in the correct range to the IF amplifier, while they are undesirable to start with.

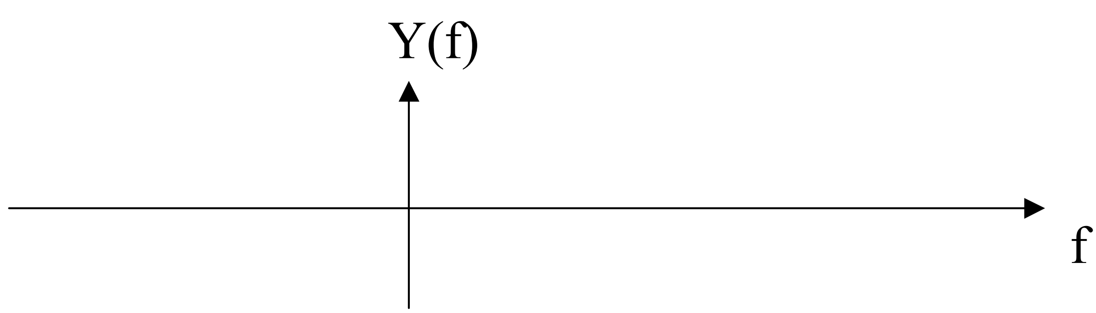
• **Question**: how to determine the image frequency which will be a problem to a specific AM signal ?

• How to use the RF amplifier to remove the image frequency problem



• **Application to Frequency Division Multiplexing**





- Demodulation