II.B Baseband Transmission (Reception & Applications)

Digital Baseband Reception
- Matched filter
  - Definition
  - Implementation (correlator)
- Application to digital baseband signal

M-ary Baseband Reception

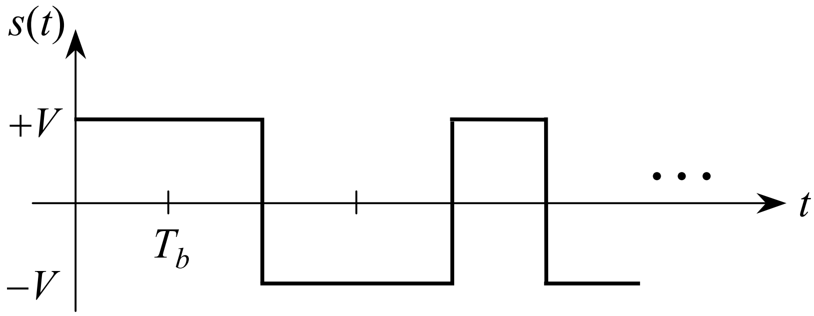
Brief Review of Probability and Noise Concepts
- Basic pdf definitions
- White noise
- Narrowband noise

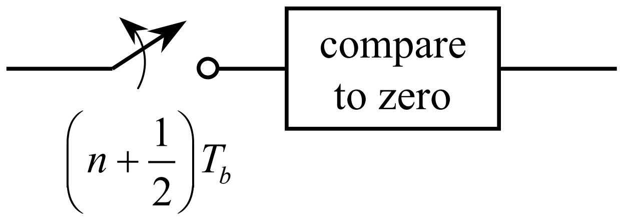
Bit Error Rate
- Error probability
- Threshold definition
- Application to the binary matched filter detector
- Examples

M-ary baseband Performance

Application to the CD Format
- Speech range
- D/A converter issues
- Oversampling
- Noise shaping & Dither effects

10) Digital Baseband Reception





- Goal: to recover $s(t)$ from potentially noisy received signal

  — use a filter to decrease the effect of noise

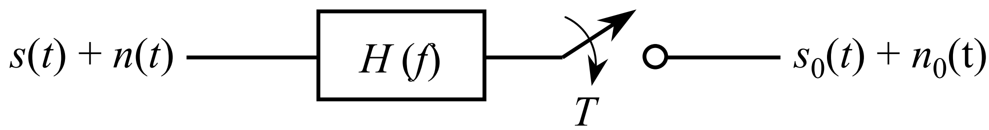
- Generic filter does not take advantage of known signal shape transmitted

  → better result obtained when using that information

  ↓

  "matched filter"

- **Matched Filter**



✶ Definition: a matched filter is a linear filter which minimizes the output signal to noise ratio (SNR) ρ at time T, where ρ is defined as:

$$\rho = \frac{s_0^2\left(T\right)}{n_0^2\left(t\right)}$$

$$\rho = \frac{\left|\int_{-\infty}^{+\infty} S\left(f\right)H\left(f\right)e^{j2\pi fT}df\right|^2}{\int_{-\infty}^{+\infty} S_n\left(f\right)\left|H\left(f\right)\right|^2 df}$$

✶ Goal: find $H(f)$ which minimizes ρ

✶ Proof: Use Schwartz's inequality which states:

$$\left|\int A\left(x\right)B\left(x\right)dx\right|^2 \le \int\left|A^2\left(x\right)\right|dx \int\left|B^2\left(x\right)\right|dx$$

equality holds only when $A(n) = KB^*(x)$
K real constant

$$\rho = \frac{\left| \int_{-\infty}^{+\infty} S(f) H(f) e^{j2\pi fT} df \right|^2}{\int_{-\infty}^{+\infty} S_n(f) \left| H(f) \right|^2 df}$$

$$\left| \int S(f) H(f) e^{j2\pi fT} df \right|^2 = \left| \int \overbrace{\frac{S(f)}{\sqrt{S_n(f)}}}^{A} \; \overbrace{H(f)\sqrt{S_n(f)}\, e^{j2\pi fT}}^{B} \; df \right|^2$$

$$\leq \int_{-\infty}^{+\infty} \left| \frac{S(f)}{\sqrt{S_n(f)}} \right|^2 df \cdot \int_{-\infty}^{+\infty} \left| H(f)\sqrt{S_n(f)}\, e^{j2\pi fT} \right|^2 df$$

$$\leq \int_{-\infty}^{+\infty} \frac{\left| S(f) \right|^2}{S_n(f)} df \cdot \int_{-\infty}^{+\infty} \left| H(f) \right|^2 S_n(f) \left| e^{j2\pi fT} \right|^2 df$$

$$\Rightarrow \quad \rho \leq \frac{\int \frac{\left| S(f) \right|^2 df}{S_n(f)} \cdot \int_{-\infty}^{+\infty} \left| H(f) \right|^2 S_n(f) df}{\int_{-\infty}^{+\infty} S_n(f) \left| H(f) \right|^2 df}$$

$$\Rightarrow \quad \rho \leq \int_{-\infty}^{+\infty} \frac{\left| S(f) \right|^2 df}{S_n(f)}$$

$P_{\max}$ obtained when $\dfrac{S(f)}{\sqrt{S_n(f)}} = KH^*(f)\sqrt{S_n(f)}\, e^{-j2\pi fT}$

$$\Rightarrow \quad \boxed{H(f) = K \frac{S^*(f)}{S_n(f)} e^{-j2\pi fT}}$$

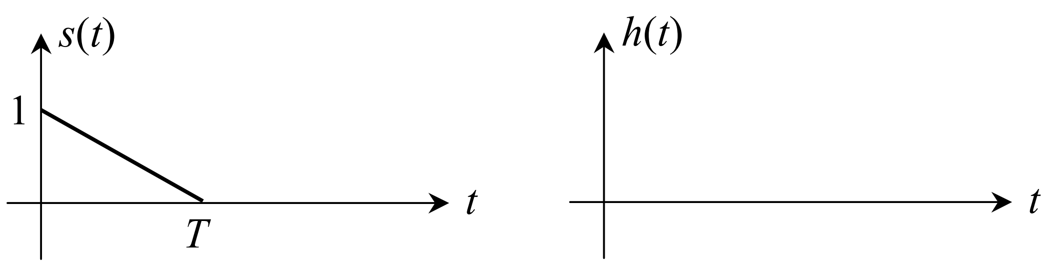
$$\text{When } n(t) \text{ is white noise} \rightarrow S_n(f) = \sigma_n^2$$

$$\Rightarrow \quad H(f) = KS^*(f)e^{-j2\pi fT}$$

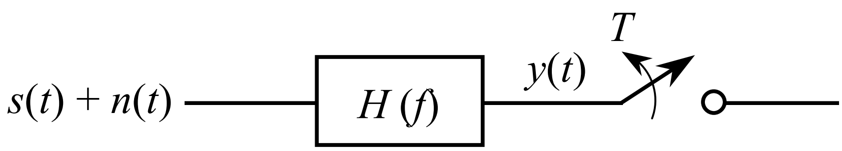
$$\Downarrow$$

$$h(t) = Ks(T-t)$$

✶ Example:



✶ Matched Filter Implementation (correlator)



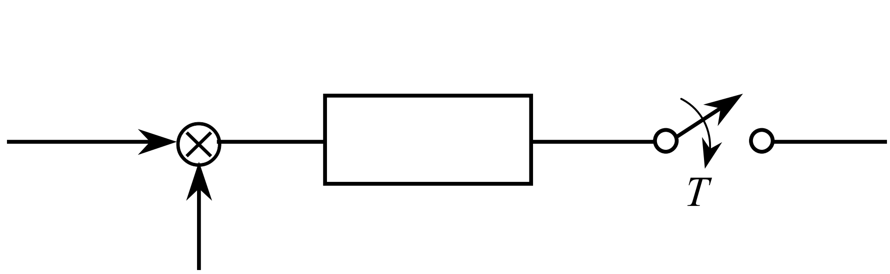
$$y(t) = \big(s(t) + n(t)\big) * h(t)$$

$$= \int_{-\infty}^{+\infty} \big(s(\tau) + n(\tau)\big) h(t-\tau)\, d\tau$$

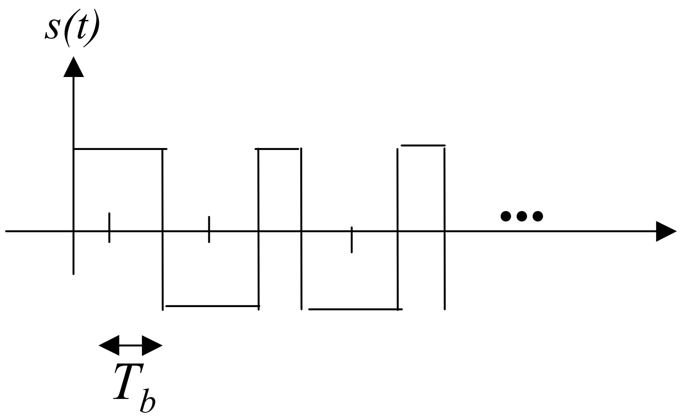
$$h(\tau) = Ks(T-\tau)$$

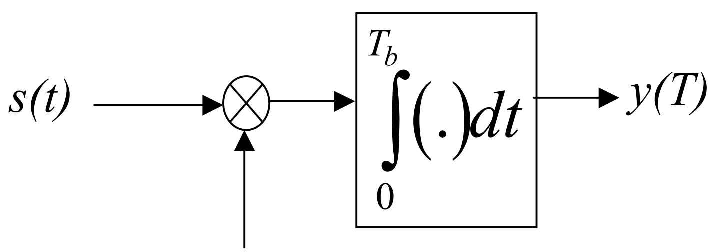
$$= \int_{-\infty}^{+\infty} \big(s(\tau) + n(\tau)\big) s(\qquad\qquad)\, d\tau$$

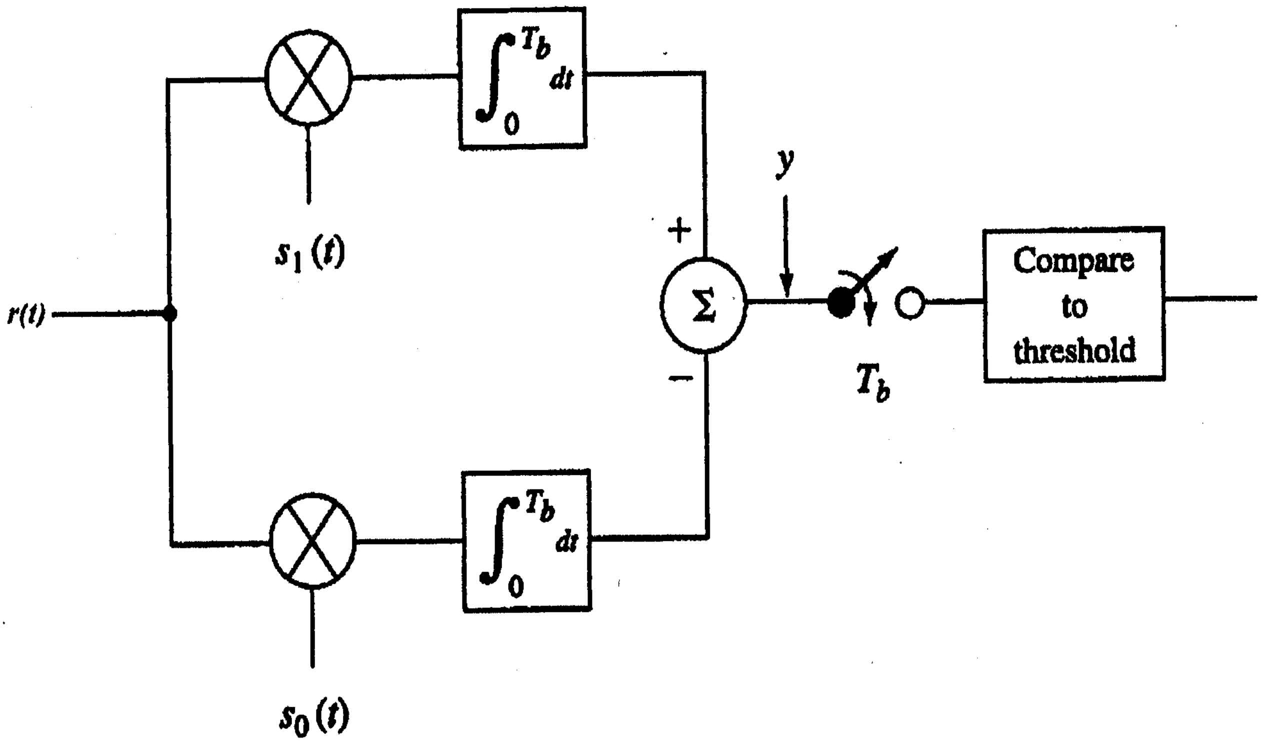
$$=$$



- Matched Filter Detector for Digital Baseband

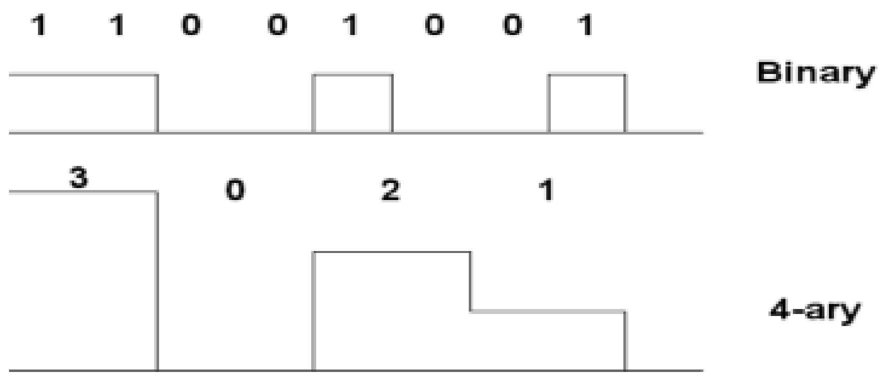






$r(t)$
$s_1(t)$
$s_0(t)$
$\int_0^{T_b} dt$
$\int_0^{T_b} dt$
+
−
Σ
y
T_b
Compare
to
threshold

- M-ary Baseband Reception
  - We can transmit more than two symbols
  - Example: 4-ary baseband communications

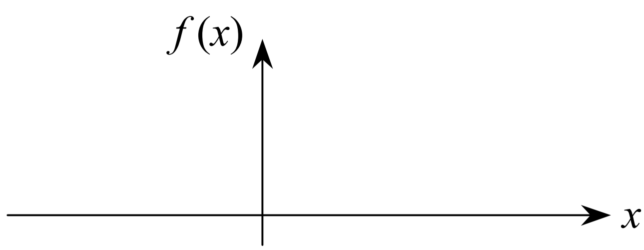


  - How to apply binary result to M-ary case ?

11) Brief Review of Probability and Noise Concepts

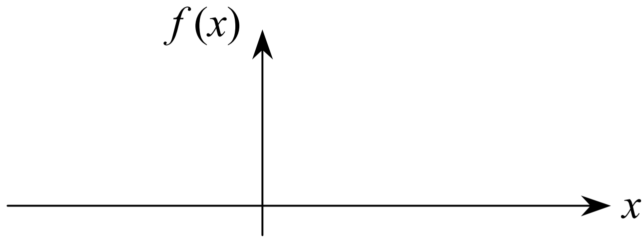
- Probability distribution function $F(x) =$
- Probability density function $f(x)$
- Expected value $E(x) =$
- Variance $\sigma_x^2 =$
- Basic pdfs

(1) Uniform pdf



$f(x) =$

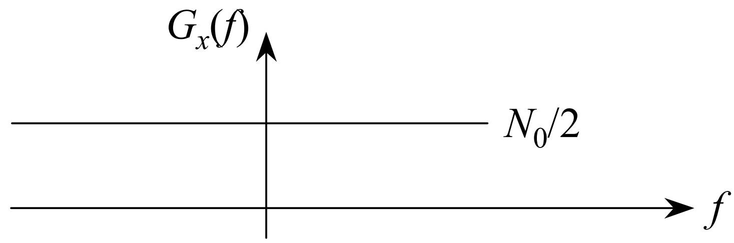
(2) Gaussian pdf



$$f(x) =$$

- White Noise

  $x(n)$ is random with a constant PSD



- Narrowband Noise

  — most communication systems contain bandpass filters

  — white noise gets transformed into BP noise

  — when noise band is small compared to center frequency f_c

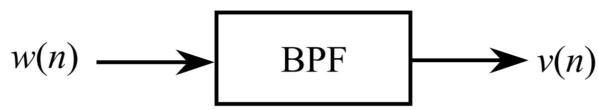
  → BP noise called narrowband noise

Quadratic noise components

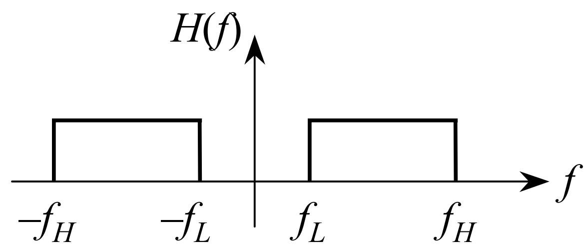
expressed as:

$$w(t) = x(t)\cos(2\pi f_c t) - y(t)\sin(2\pi f_c t)$$
$$= \mathrm{Re}\left[r(t) e^{j2\pi f_c t} \right]$$

complex function



white noise
$N\left(0,\sigma_w^2\right)$



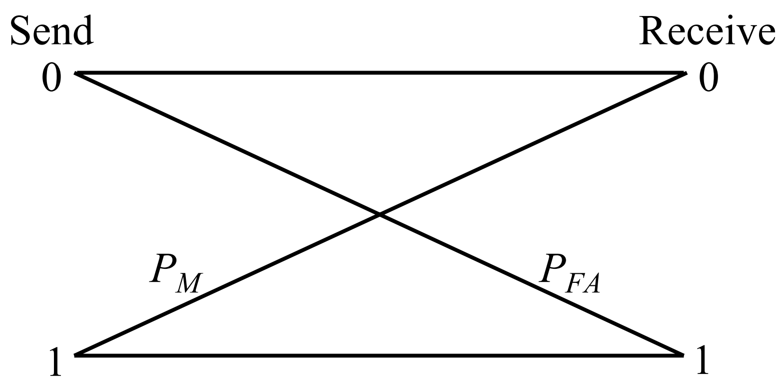
$$P_{\text{out}} =$$

$$E\left[V\left(n\right)\right]=$$

$$\sigma_V^2 =$$

12) Bit Error Rate

- Evaluate errors made during transmission of hits



- Errors occur when:

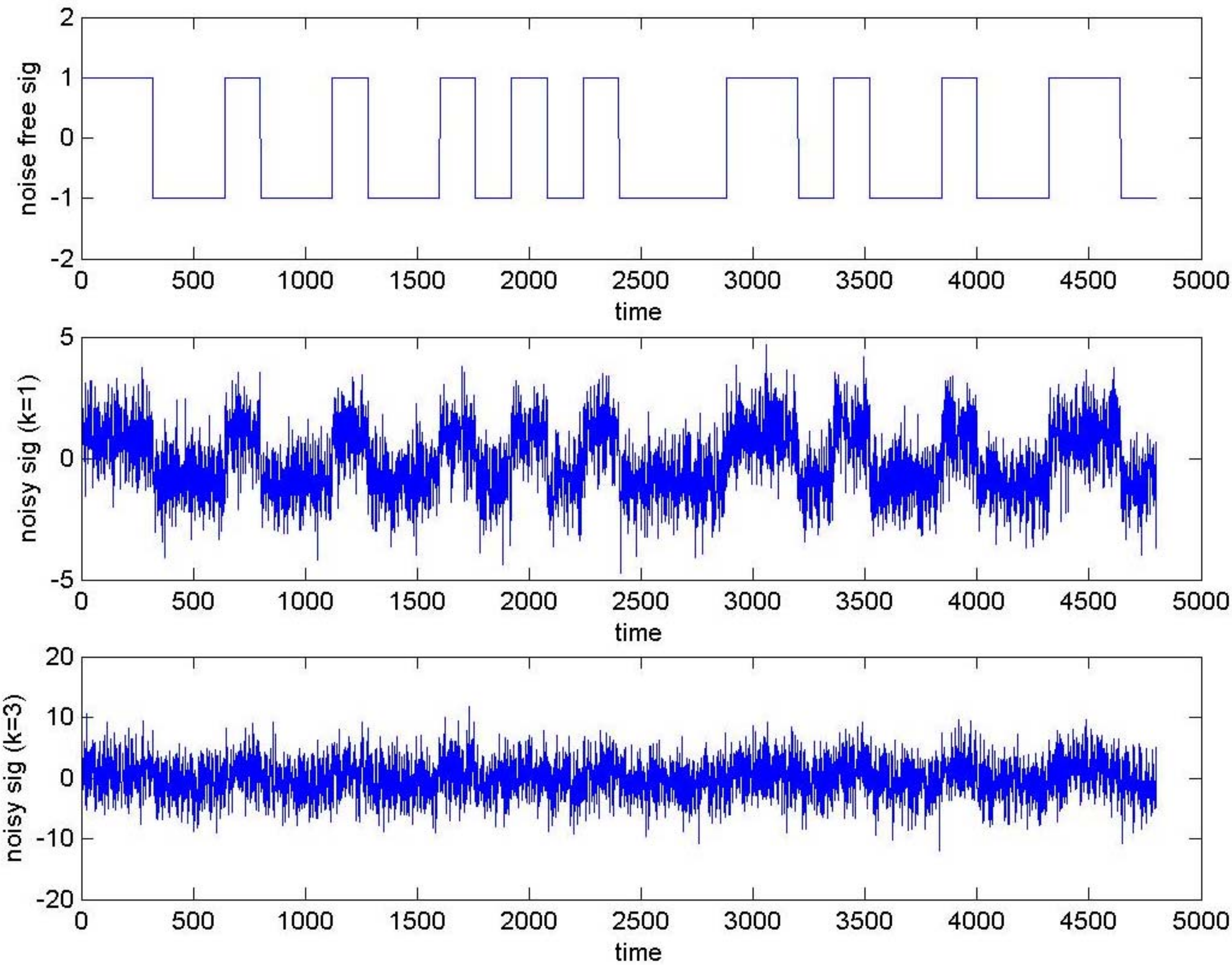
receive "1" when "0" is sent
receive "0" when "1" is sent

- How to decide if you receive a "1" or "0" when transmission is noisy ??

$\longrightarrow$ ***detection theory***

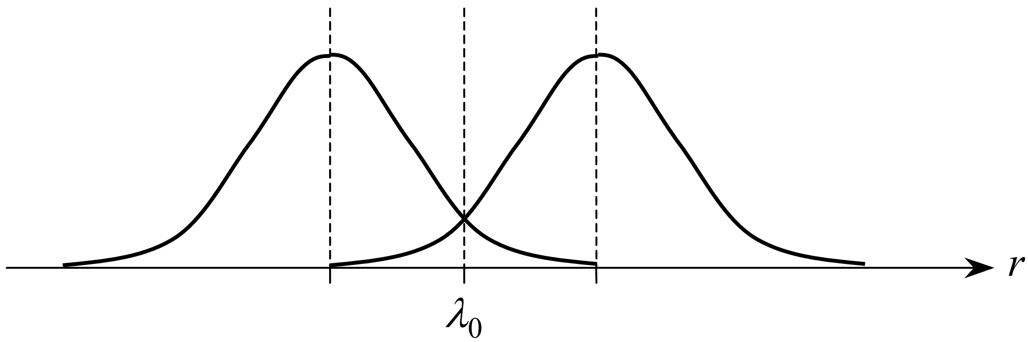
$$r(t)=s(t)+Kw(t)$$

$N(0,\sigma_w^2=1)$



• Assume you have additive white noise distortion at the receiver

$$r(t)=s_i(t)+w(t),\ \ s_i(t)=s_0\ ,\ s_1$$



- $f(w) =$

- $f(r|s_0) =$

- $f(r|s_1) =$

For which range of "r" do you decide you sent a	
"1" (s_1)	"0" (s_0)

How to pick λ_0 ?

- Goal: To quantify likelihood of making an error in assigning bit values at the receiver.

Assume transmission is noisy

$$r(n) = s(n) + w(n)$$

received signal

signal sent $\begin{cases} s_0 \\ s_1 \end{cases}$

transmission noise (random Gaussian) $N(0, \sigma_w^2)$

- Statistics of $r(n)$

  – $r(n)$ $\begin{cases} \text{deterministic} & ? \\ \text{random} & ? \end{cases}$

  – mean of $r(n)$?

- Notation

H_1: receive a "1" (s_1)

H_0: receive a "0" (s_0)

- Correct decisions:

$$P\left(H_1 \middle| s_1\right) =$$

$$P\left(H_0 \middle| s_0\right) =$$

- Incorrect decisions:

$$P\left(H_1 \middle| s_0\right) =$$

$$P\left(H_0 \middle| s_1\right) =$$

- Overall probability of error:

$$P_e =$$

- P_e when there is equal probability of sending s_0 & s_1

$$P_e = 2\frac{1}{2}P(H_1 \mid s_0) = P(H_0 \mid s_1) = \int_{\lambda_0}^{\infty} f(r \mid s_0)dr$$

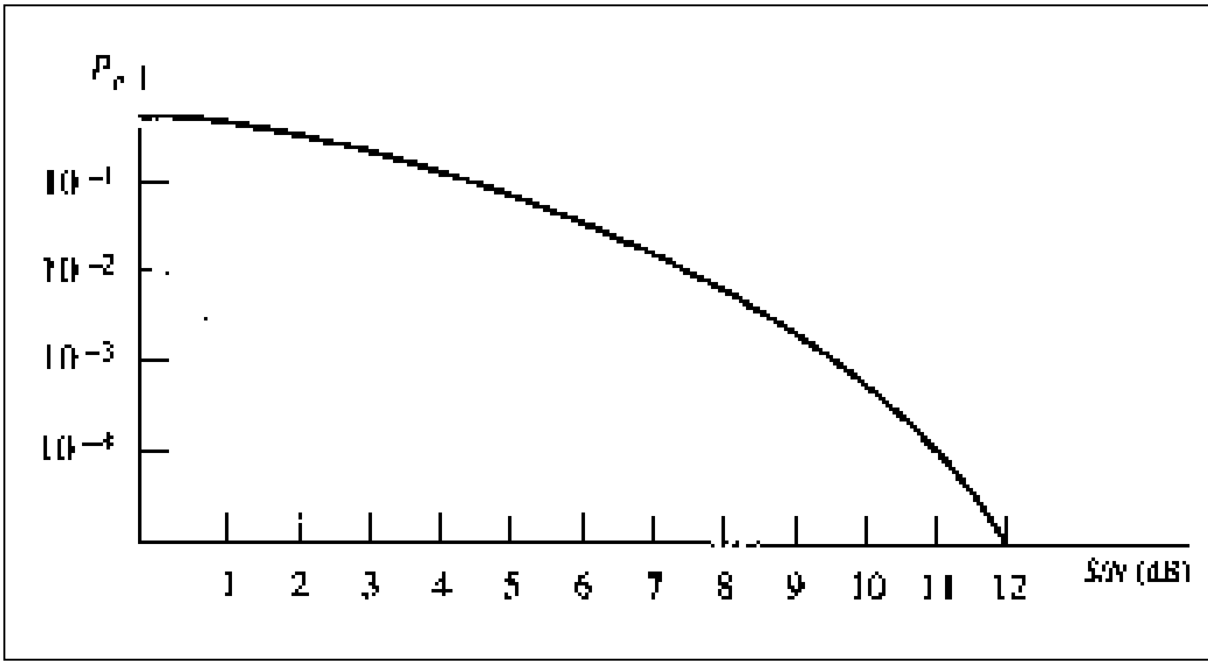
- Assume s_0=-V & s_1=+V

- Definitions: Q, erf, & erfc functions

$$Q(x) = \int_x^{\infty}\frac{1}{\sqrt{2\pi}}e^{-u^2/2}du = \frac{1}{2}\text{erfc}\left(\frac{x}{\sqrt{2}}\right)$$

$$\text{erfc}(x) = 1 - \frac{2}{\sqrt{\pi}}\int_0^x e^{-u^2}du = 1 - \text{erf}(x)$$

BER for single sample detector



- How to select the threshold λ_0 ?

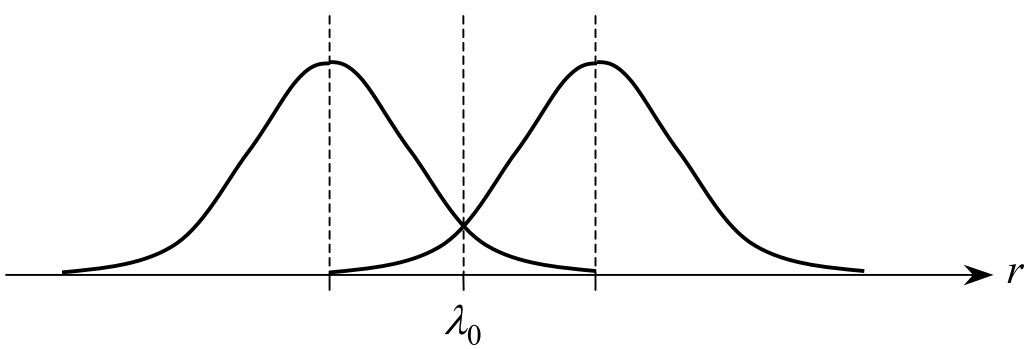
⟶ Maximum likelihood detector approach

Minimize the overall probability of error P_e

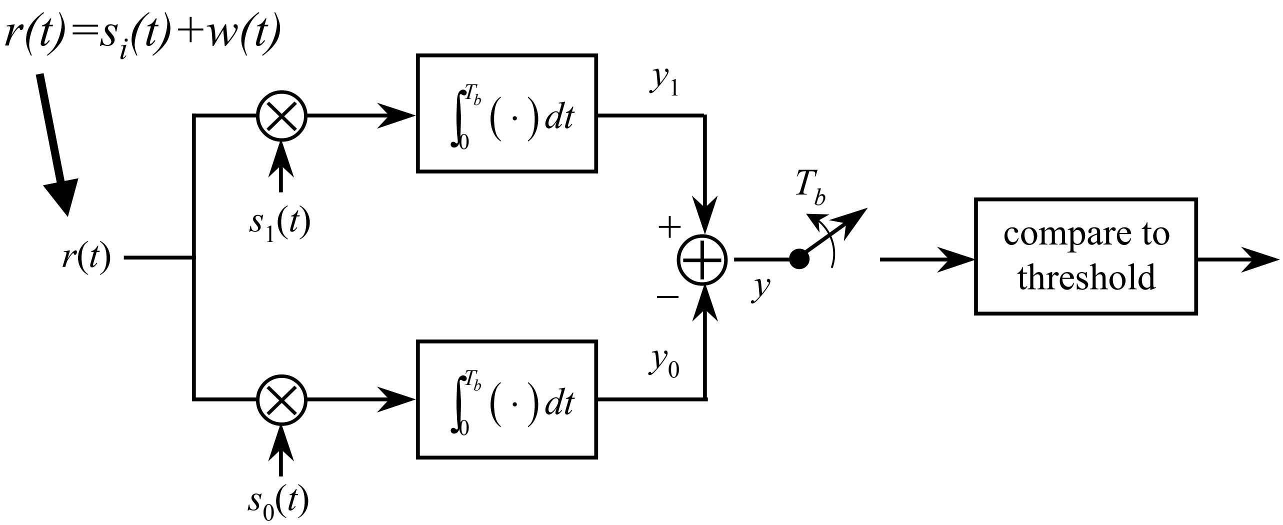
$$P_e = P(H_1 \mid s_0)P(s_0) + P(H_0 \mid s_1)P(s_1)$$

$$\lambda_0 = \frac{s_0 + s_1}{2} + \text{In}\left(\frac{P_0}{P_1}\right)\frac{\sigma_w^2}{s_1 - s_0}$$

(More details in EC4570…)



- Application to Binary Matched Filter Detector



Is y random or deterministic?

- Need statistics on $y = y_1 - y_0$

- pdf of y: ?

$$y_1 = \int_0^{T_b} r(t)\, s_1(t)\, dt$$

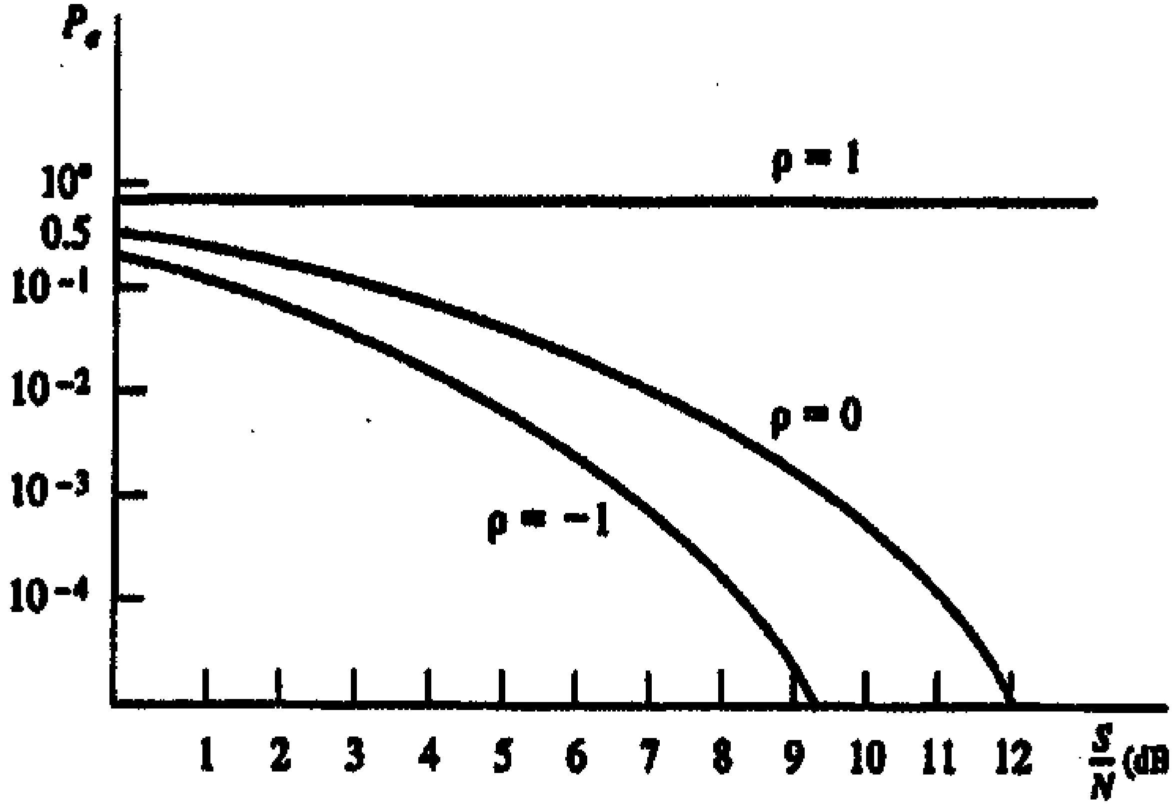
$$= \int_0^{T_b} \left(s_i(t) + w(t)\right) s_1(t)\, dt \qquad s_i(t) = s_1(t) \text{ or } s_0(t)$$

$$= \int_0^{T_b} s_i(t)\, s_1(t)\, dt + \underbrace{\int_0^{T_b} w(t)\, s_1(t)\, dt}$$

$$y_0 =$$

$$y = y_1(t) - y_0(t)$$

$$=$$



Bit error rate for matched filter detector

$$\rho = \frac{\int_0^{T_b} s_0(t) s_1(t) dt}{E}$$

$$E = \frac{1}{2}\left(\int_0^{T_b} s_0^2(t) dt + \int_0^{T_b} s_1^2(t) dt \right)$$

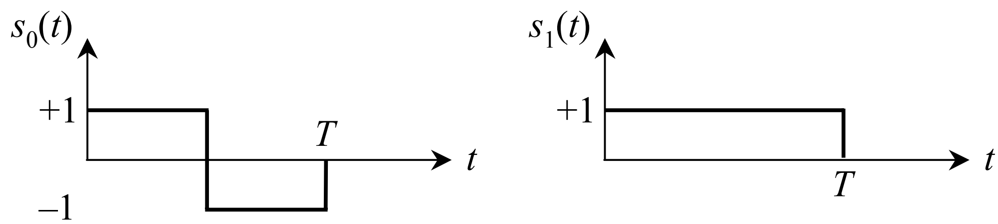
- How to compute the threshold λ_0 ?

Recall for simple detector, the threshold was selected as the mid point between the two means for basic problem.
How can we apply the result here ?

$E[y|s_0]=$

$E[y|s_1]=$

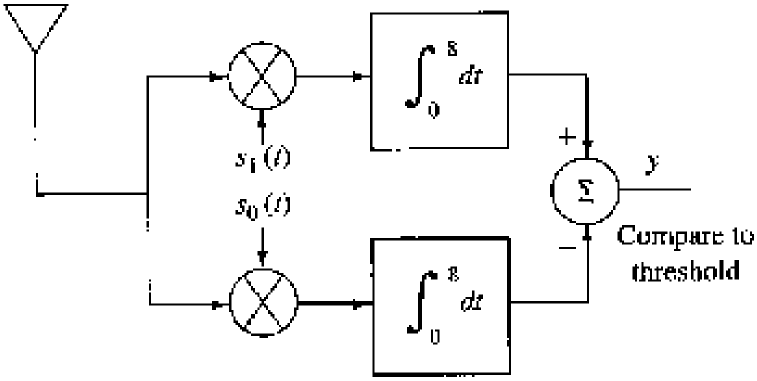
- Example



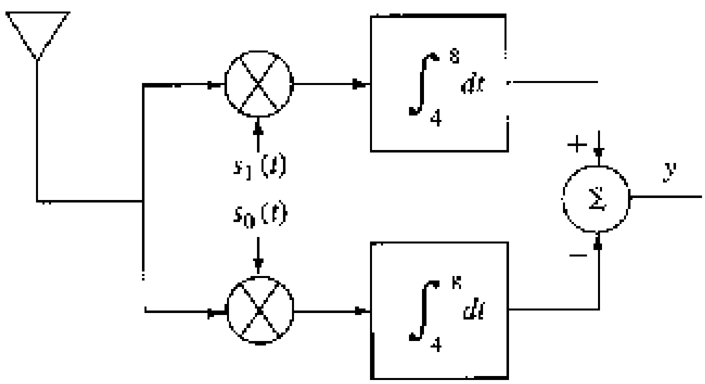
$T = 8\ 10^{-3}\ s$

- Design a matched filter detector for the two signals.

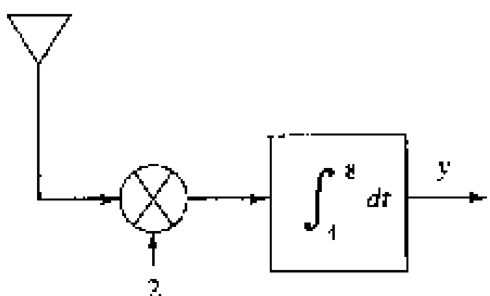
- Find P_e when the additive white noise has a power $P = 10^{-3}$ w/Hz.


$\int_0^8 dt$
$\int_0^8 dt$
$s_1(t)$
$s_0(t)$
+
−
Σ
y
Compare to threshold

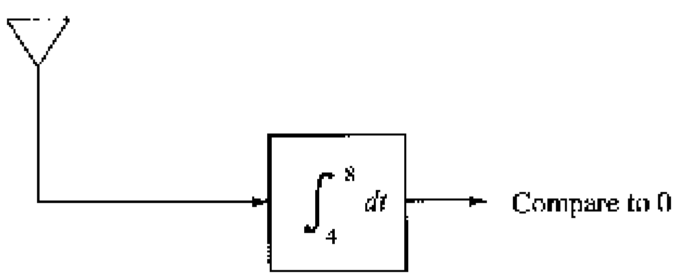
(a)


$\int_4^8 dt$
$\int_4^8 dt$
$s_1(t)$
$s_0(t)$
+
−
Σ
y

(b)

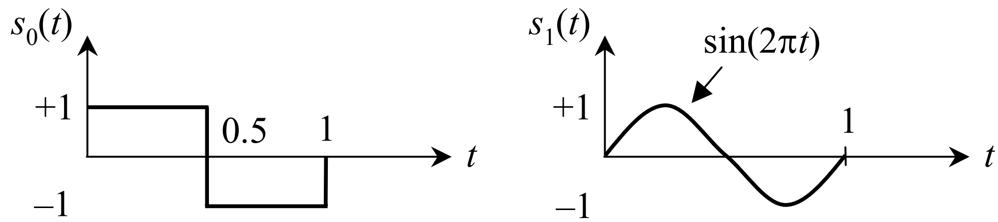

$\int_4^8 dt$
2
y

(c)


$\int_4^8 dt$
Compare to 0

(d)

- Example



- Design a matched filter detector for the two signals.

- Find P_e when the additive white noise has a power $P_e = 0.1$ w/Hz.

- M-ary Baseband Performance

Assume we send M = 4 different levels (B_i= 0, A, 2A, 3A, i=0,…3)

$$r(t) \longrightarrow \boxed{\int_0^{T_b} (\bullet)dt} \longrightarrow \textbf{?}$$

Assume additive Gaussian noise

$$r(t)=s(t)+n(t)$$

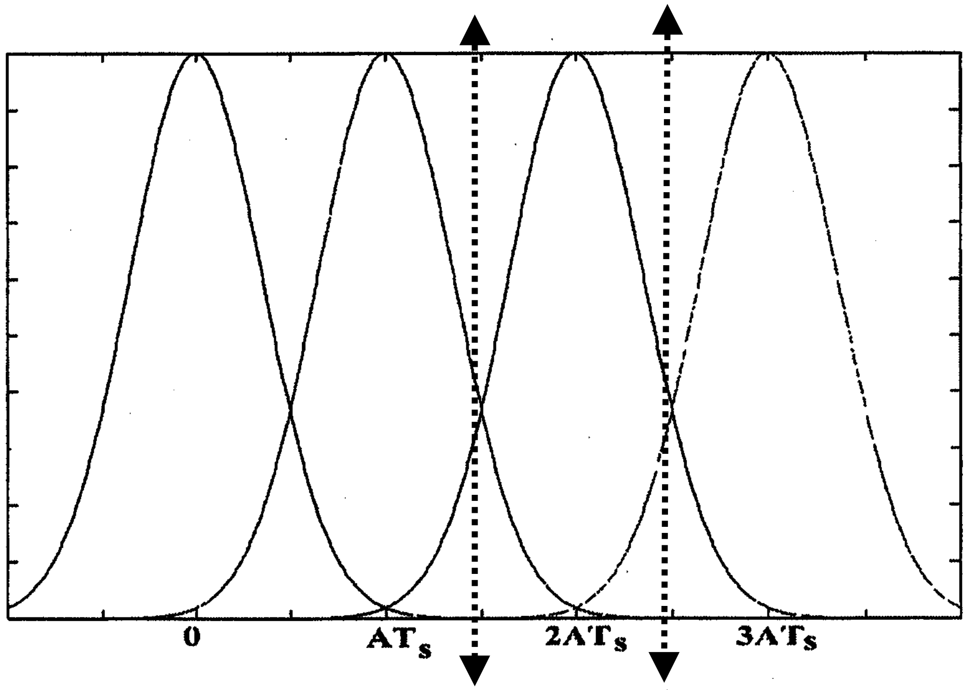


Figure 2.112 - Conditional Probabilities for M-ary Baseband

- P(error in receiving B_2)=

• P(error in receiving B_2) $= 2Q\left(\sqrt{\frac{A^2T_s}{2N_0}}\right) = erfc\left(\sqrt{\frac{A^2T_s}{4N_0}}\right)$

• P(error in receiving B_1)=

• P(error in receiving B_0)=

• P(error in receiving B_3)=

• Assume each error may occur as likely as the others

⇒ P_e=

• Assume we send M different levels, compute the overall probability of error becomes:

13) Application: Compact Disk

✶ Dynamic range for audio signals
Ref [3]

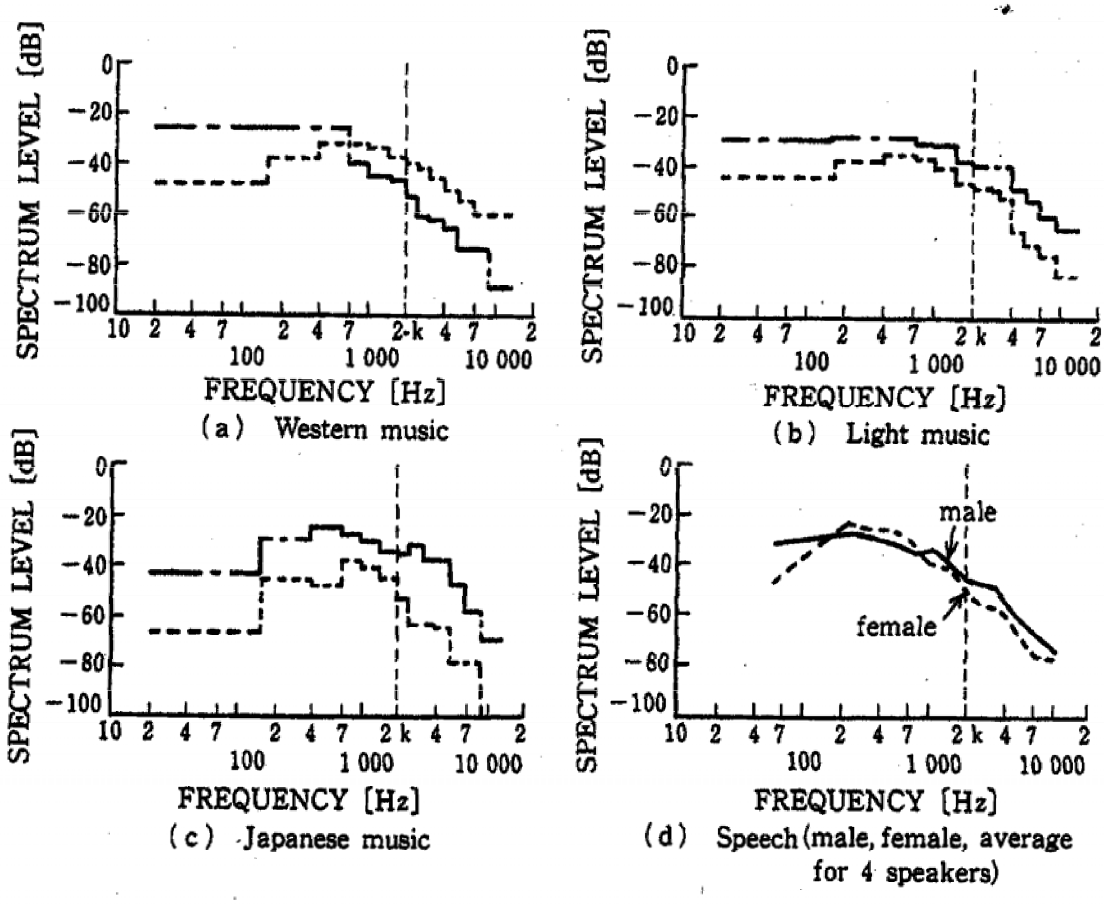
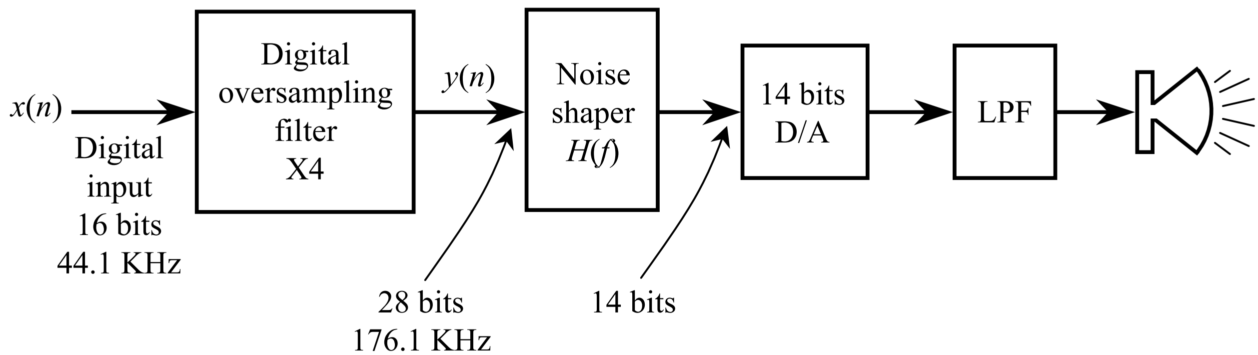
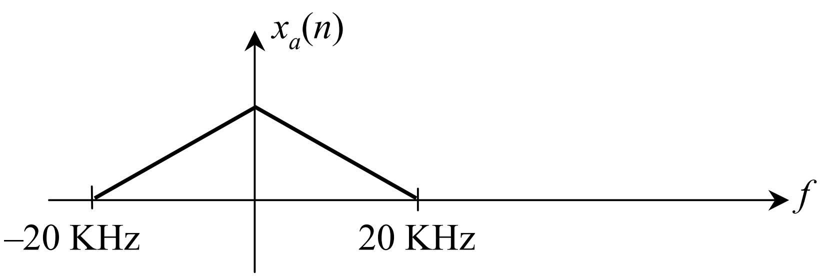
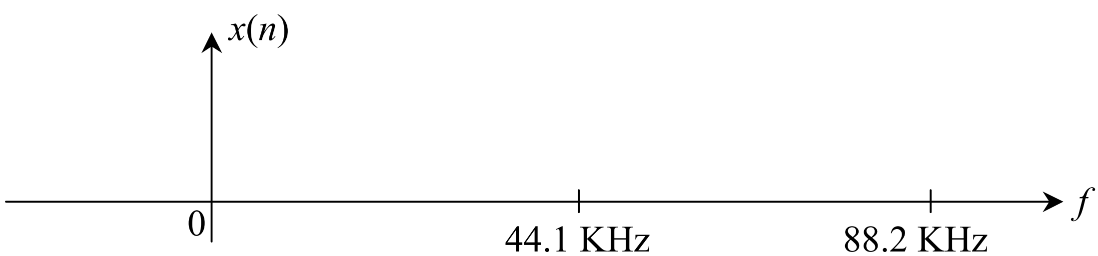


Fig. 2.7 Average spectra of various types of broadcast programs.

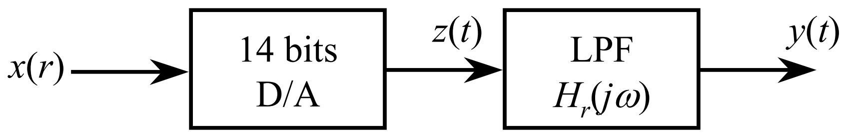
✶ Signal reproduction in CD Player

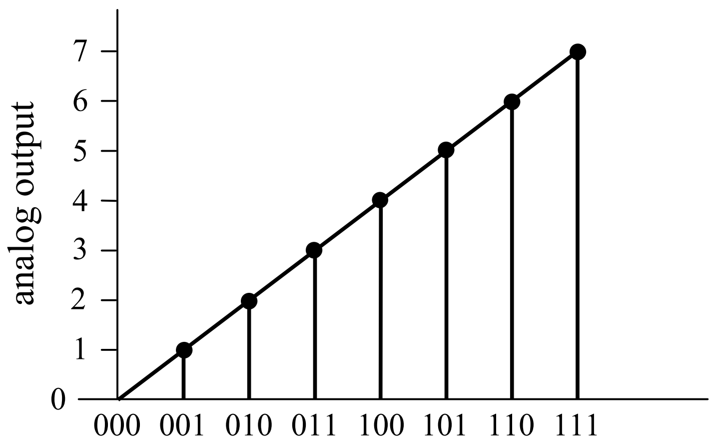




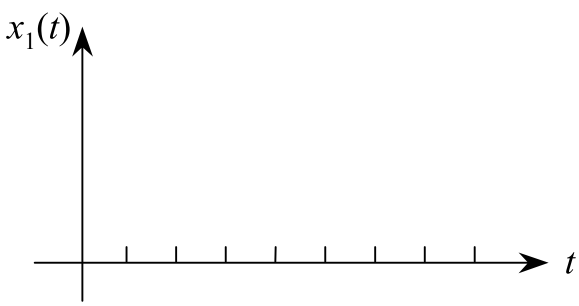


- Why use oversampling ?
- First, assume we don't use oversampling.

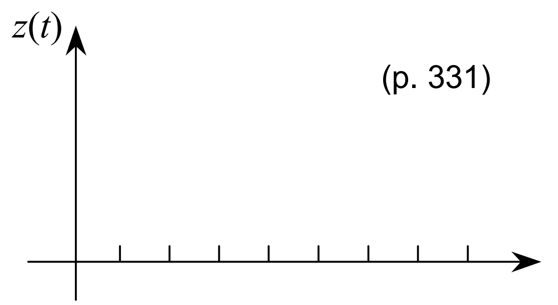




Input/output relationship for a unipolar D/A (3 bits) converter.

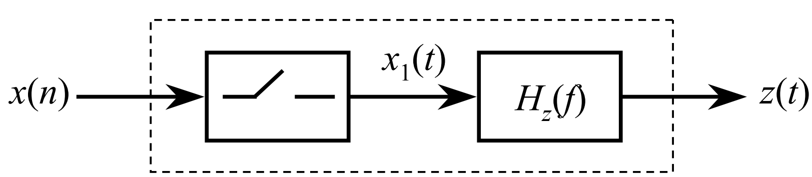


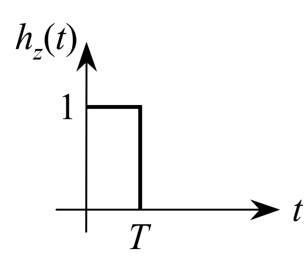
Ideal D/A Converter



(p. 331)

Practical D/A Converter





✶ D/A Converter Output Expression

$$x_1(t) = \sum_k x(k)\delta(t - kT)$$

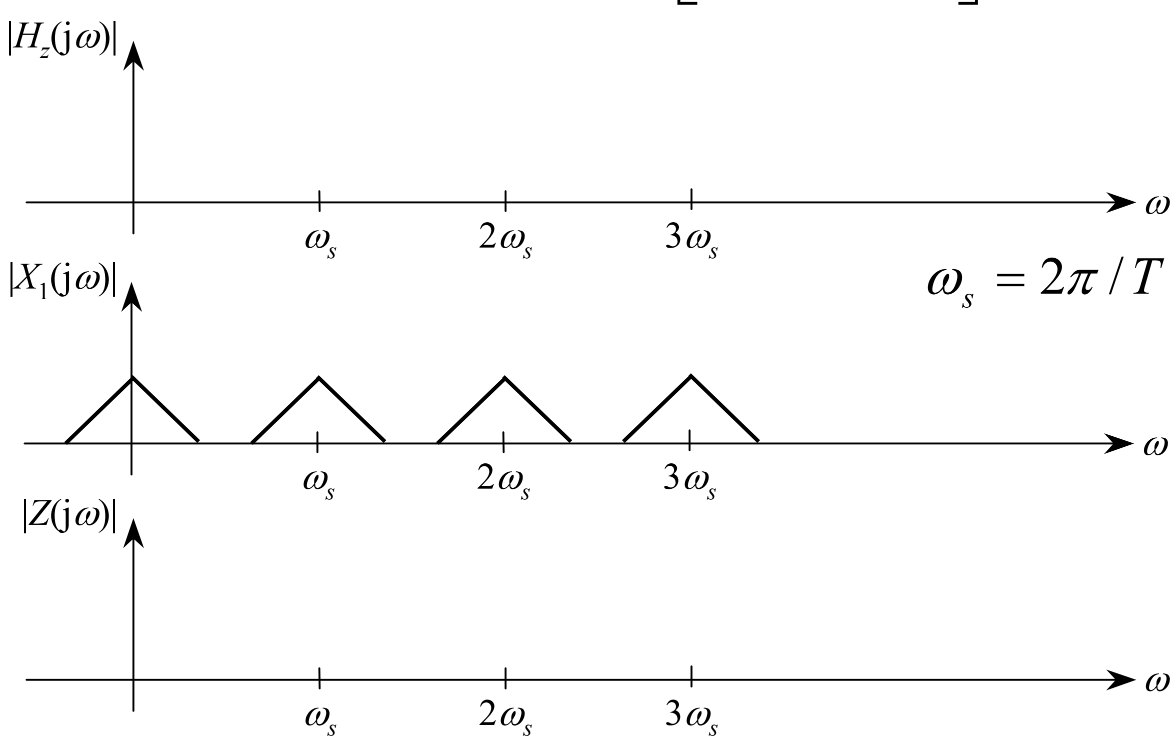
$$z(t) = x_1(t) * h_z(t)$$

$$\rightarrow Z(j\omega) = X_1(j\omega) H_z(\omega)$$

$$H_z(\omega) = \int_0^T e^{-j\omega t} dt$$

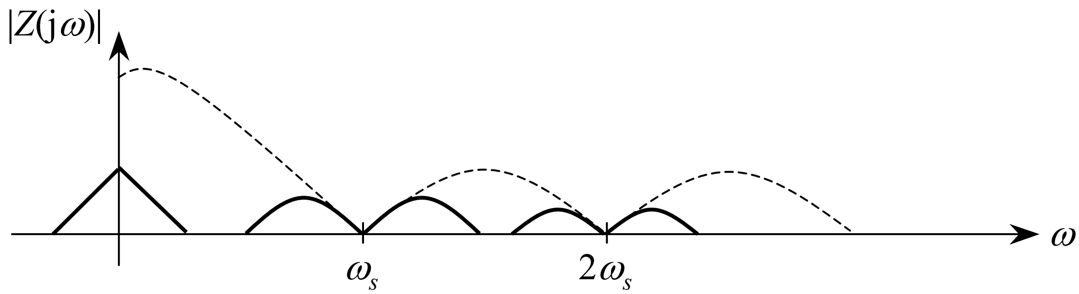
$$= \frac{1}{-j\omega}\left(e^{-j\omega T} - 1\right)$$

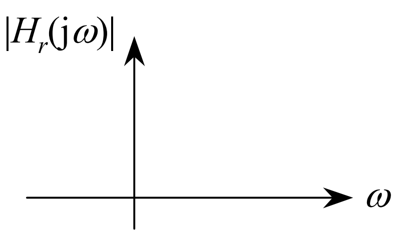
$$= e^{-j\omega T/2}\left[\frac{\sin(\omega T/2)}{\omega/2}\right]$$

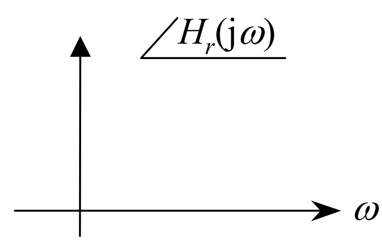


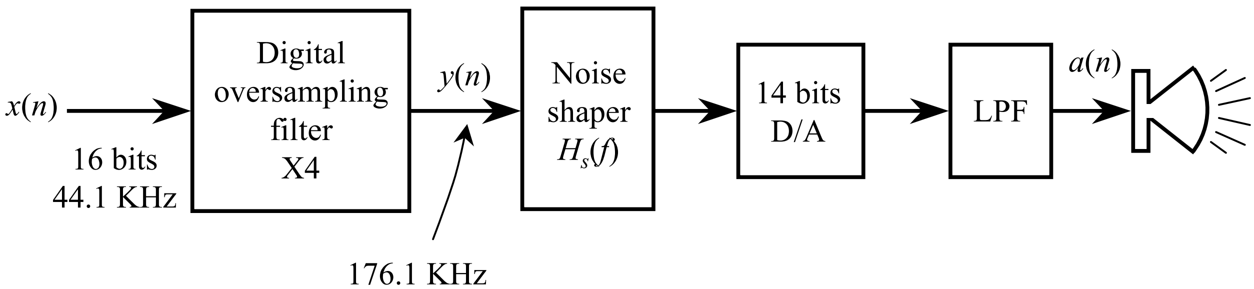
$$\omega_s = 2\pi / T$$

- Need for LPF filter ?
  - to smooth out output steps
  - to undo distortion added by D/A converter

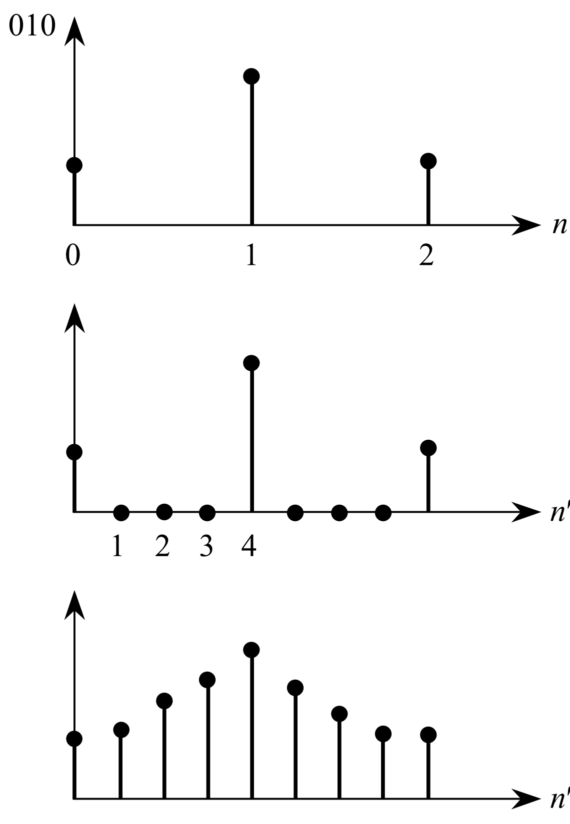








- Oversampling in the Time Domain

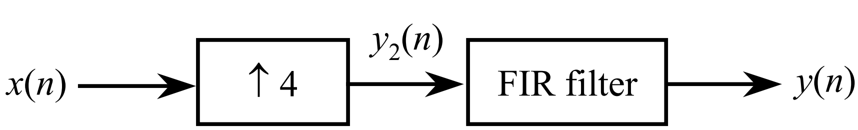


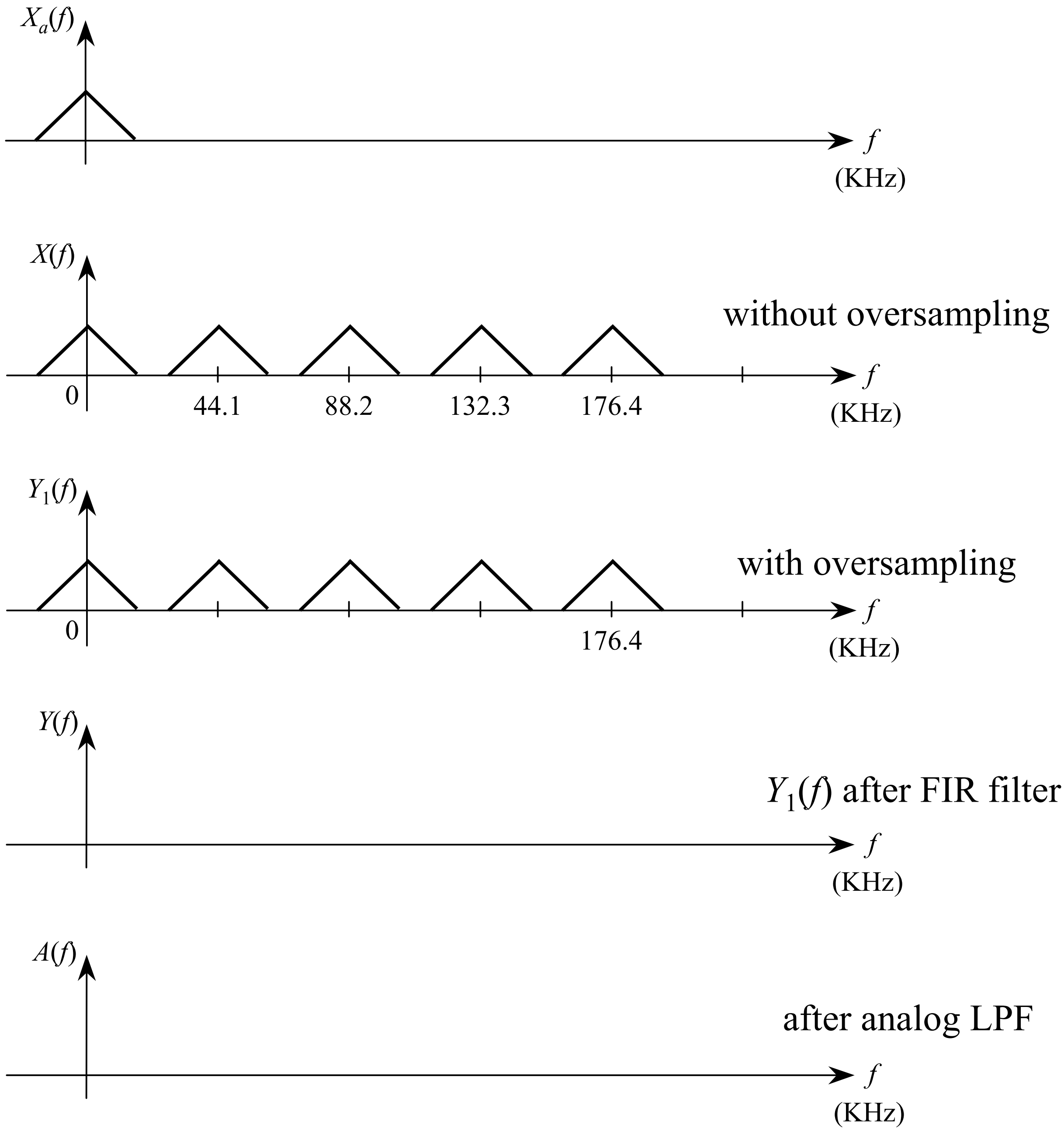
input to upsampler by 4

output to upsampler by 4

output of FIR filter

$$001 \quad 010 \quad \Rightarrow \quad 001 \; \underbrace{000 \quad 000 \quad 000} \; 010$$





★ Advantages of Oversampling

- Example:

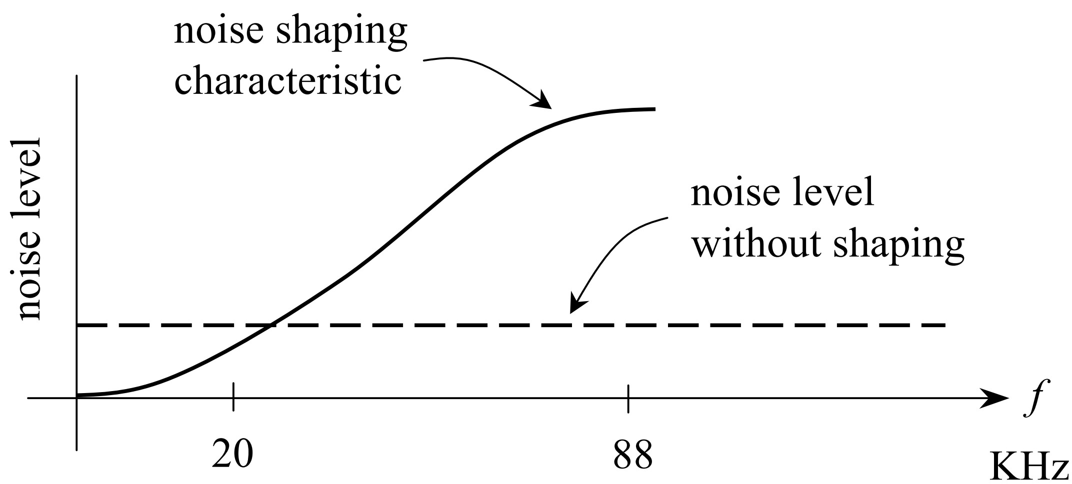
Assume
1) you have an analog signal which spans [0 20KHz],
2) the D/A converter has a sampling frequency f_s=176.4KHz.
Determine the characteristics (order and cutoff frequency) for the anti-imaging Butterworth type filter which satisfy the following specifications:

1) Image frequencies must be attenuated by at least 40dB
2) Signal components may be altered by a maximum of 0.5dB

$$H(f) = \frac{1}{\left[1 + \left(f / f_c\right)^{2n}\right]^{1/2}}$$

✶ Noise Shaping Filter

— Goal: to decrease noise in the audio band



✶Dither

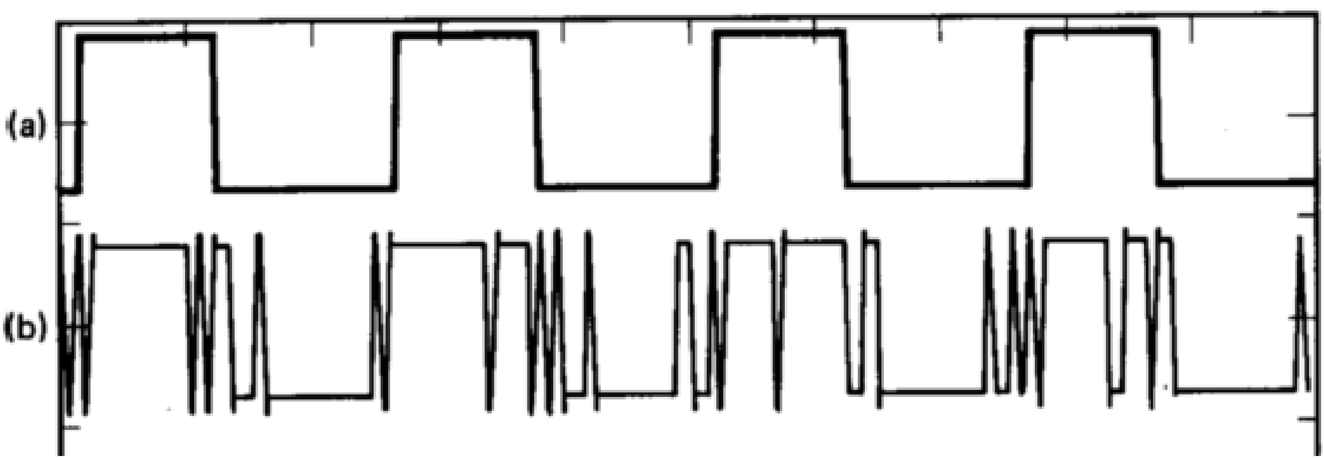


Figure 2.121 - Coding of Dithered Signal

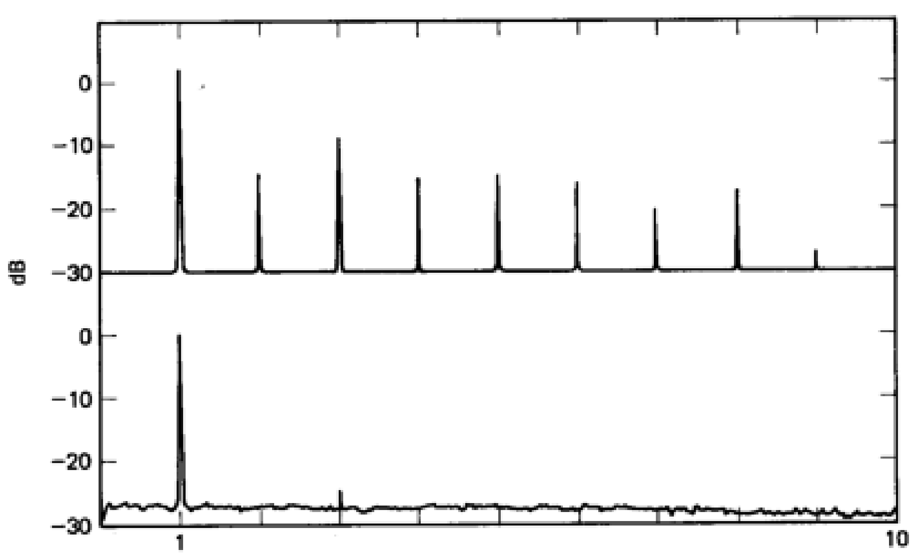
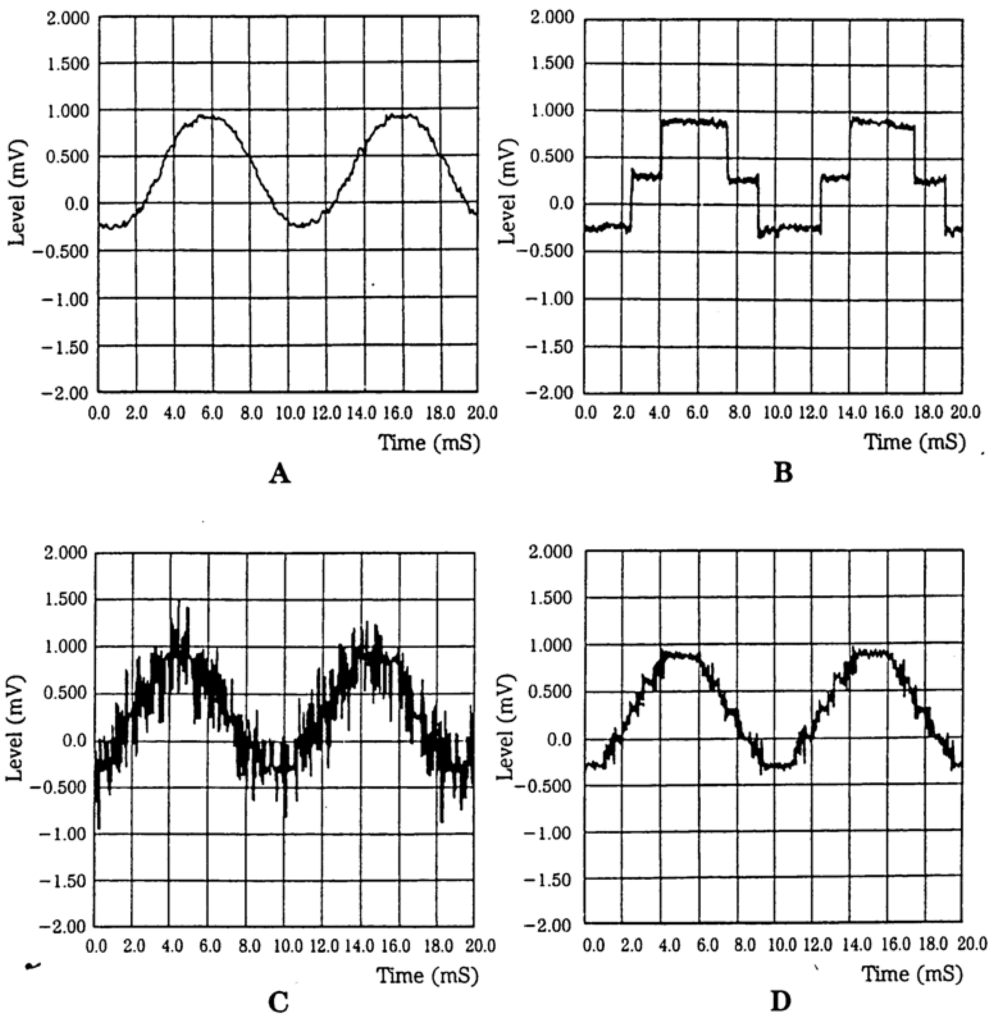


Figure 2.122 - Fourier Transform of Dithered Signal

✶Dither & noise shaping effects



Poh1man Fig. 6-27 An example of noise shaping showing a 1 kHz sinewave with -90 dB amplitude; measurements are made with a 16 kHz lowpass filter.

A. Original 20 bit recording.

B. Truncated 16 bit signal.

C. Dithered 16 bit signal.

D. Noise shaping preserves information in lower 4 bits.

Ref [4,5]

CD Section References:

[1] S. Mitra, *Digital Signal Processing*, McGraw-Hill, 1998.
[2] K. Pohlmann, A. Red, *Compact Disk Handbook*, 2, 1992.
[3] H. Nakajima and H. Ogawa, *Compact Disk Technology*, IOS Press, 1992.
[4] B. Evans, Real Time Digital Signal Processing Lab, Fall 2003 (lecture 10 notes).
[5] K. Pohlmann, *Principles of Digital Audio*, McGraw-Hill, 1995