II. Introduction to Systems

- System definition and interconnections
- System properties
  - Memory
  - Causality
  - Invertibility
  - Stability
  - Time-Invariance
  - Linearity
- How to relate system Input & Output
  - Convolution sum
  - Graphical convolution
  - Convolution integral
- Convolution properties
  - Commutative
  - Associative
  - Distributive
  - Derivative of convolution
- Convolution properties applied to LTI systems
- Unit step response of a LTI
- Stability check of LTI systems
- Crosscorrelation

II. Introduction to Systems

1) What are systems?

- Any process that results in the transformation of a signal
- Block diagram is used to represent a system

2) How to interconnect systems?

- Series (cascade) interconnection
- Parallel interconnection

3) System Properties

- Memory

a system is memoryless if the output $y(t)$ depends only on $x(t)$ at time t.

Example: resistance, capacitor

Example:

$\boxed{\bullet \text{ Causality}}$

→ a system is causal if the output $y(t)$ at anytime depends upon <u>present and past values</u> of the input

Example: $y(t) = A\,x(t)$

$A\,x(t+1)$

$A\,x(t-1)$

$\boxed{\bullet \text{ Invertibility}}$

→ a system is invertible if distinct inputs lead to distinct outputs

Example: $y(t) = 2\,x(t+1)$

$3\,x^2(t)$

- $\boxed{\text{Stability}}$

→ a system is stable if applying a bounded input to the system leads to a bounded output

Example:

$$y(t) = 3x(t)$$

$$= \int_0^t x(\tau)\,d\tau$$

$$= \frac{dx(t)}{dt}$$

$$\cos(2t)x(t)$$

$$tx(t)$$

$$e^{x(t)}$$

- Time-Invariance

→ a system is time-invariant if a shift in the input produces the same shift in the output

$$x(t) \longrightarrow \boxed{\;S\;} \longrightarrow y(t)$$

Example:

$$\begin{aligned} y(t) &= \frac{d}{dt}x(t) \\ &= 2x(t) \\ &= x(2t) \end{aligned}$$

• Linearity

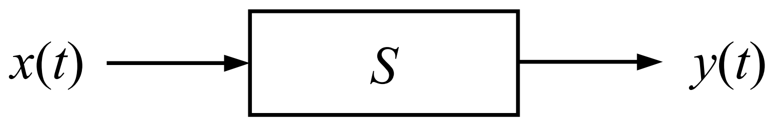
a system is linear iff the response to a weighted sum of signals is the weighted sum of the responses to each signal

$$x(t) \longrightarrow \boxed{S} \longrightarrow y(t)$$

Example:

$$\begin{aligned} y(t) &= 2x(t) \\ &= x^2(t) \end{aligned}$$

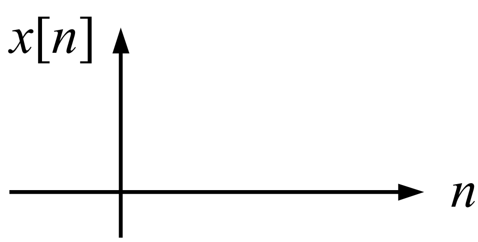
4) How to Relate Input/Output of a System



- For linear time-invariant system the unit impulse response can be used to relate input and output

4.1) Convolution Sum (for discrete time LTI system)

use property of $\delta[n]$ to compute the output y[n]



- Call h[n] response of the system to δ[n]

----- $h_k[n]$ ---------------------------- δ[n-k]

$$x[n] = \sum_{k=-\infty}^{+\infty} x[k]\delta[n-k]$$

$$y[n] =$$

- If system is TI: S{δ[n]}=

  S{δ[n-k]}=

- Examples: compute y[n] for the following inputs and impulse responses to LTI stystems

$$x[n] = 0.5^n u[n], h[n] = u[n]$$

$$x[n] = 2^n u[-n], h[n] = u[n]$$

$$x[n] = a^n, h[n] \text{ generic}$$

x[0]=0.5, x[1]=2, h[0]=h[1]=h[2]=1, all others 0

4.2) Graphical Convolution

4.3) Convolution Integral (for continuous time LTI)

5) Properties of Convolution

(1) Commutative

(2) Associative

(3) Distributive

(4) Derivative of Convolution

6) Convolution Properties Applied to LTI Systems

8) Unit Step Response of a LTI

- Useful when the impulse response of a system is difficult to find

9) Stability Check of LTI Systems Using the Impulse Response

10) Crosscorrelation

- Recall: convolution defined as:
- Replace $x(t-\tau) \rightarrow x(\tau - t)$

$R_{xy}(t)=$

Crosscorrelation tells something about similarities between two signals x(t) and y(t)

• Example: radar pulse x(t) is sent out
y(t)=x(t-T) represents the radar return

$R_{xy}(t)=$

Boy and Jenkins Time Series Analysis

Crosscorrelation

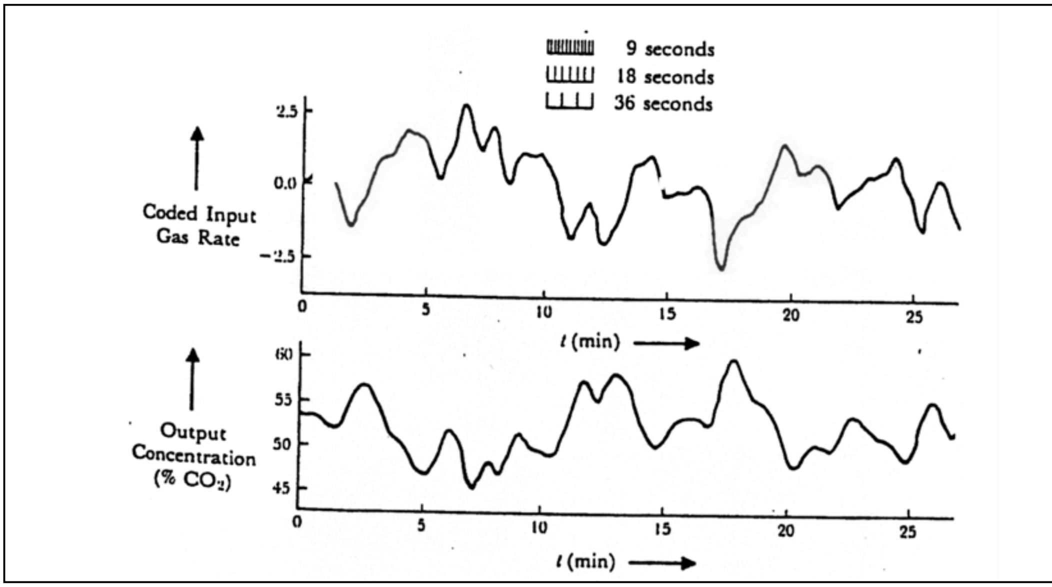


Figure 1. Input gas rate and output CO_2 concentration from a gas furnace.

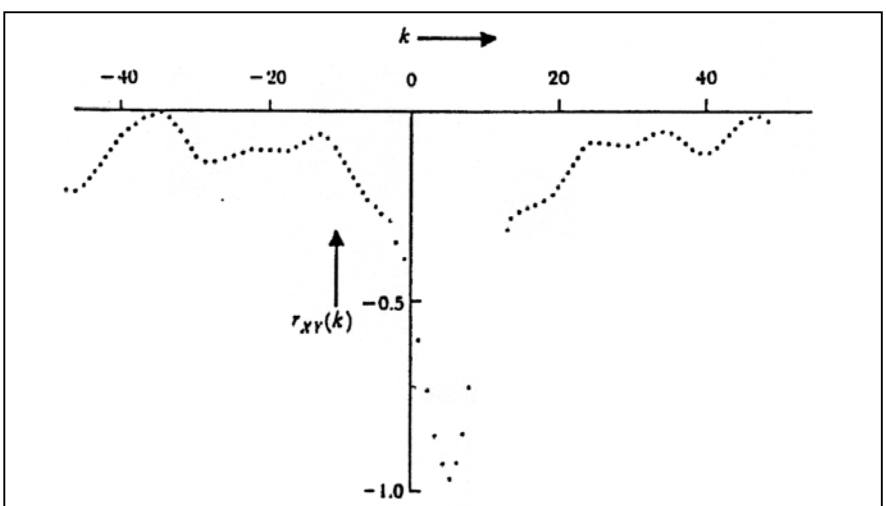


Figure 2. Cross-correlation function between input and output for coded gas furnace data read 9 second intervals.