# I. Signals & Systems
# Basic Concepts

## Complex variables review
- Definitions
- Euler's formulas
- Complex roots

## Continuous/discrete signals
- Definitions
- Energy/Power

## Signal transformations
- Time-shift
- Compression/expansion

## Periodic/aperiodic signals

## Even/Odd signals

## Basic signal types
- Real & complex exponentials
- Sinusoidal signals
- Continuous/discrete time unit impulse and step function
- Impulse function properties (sifting, scaling)
- Sinc Function

# I. Signals & Systems
## Basic Concepts

### 1) Review of Complex Variables

- $z = x + jy$

- Rectangular/polar representation

- How to go from rectangular to polar coordinates

- Complex conjugate concept

- Addition/subtraction of complex numbers

- Multiplication/division of complex numbers

- Euler’s formulas

- Complex roots

- Examples

# 2) Continuous/Discrete Signals

- Continuous signal


- Discrete signal


- Energy/power quantities

  *1) over a finite interval*

| Continuous signal | Discrete signal |
|---|---|
| | |

*2) over infinite interval*

| Continuous signal | Discrete signal |
|---|---|
| | |

• Definition: A signal is said to be a **power signal** if it has finite average power, i.e., $P_\infty < \infty$

• Definition: A signal is said to be an **energy signal** if it has finite average power, i.e., $E_\infty < \infty$

- Examples

  $x[n] = 1$, all $n$'s

  $x(t) = 2t$

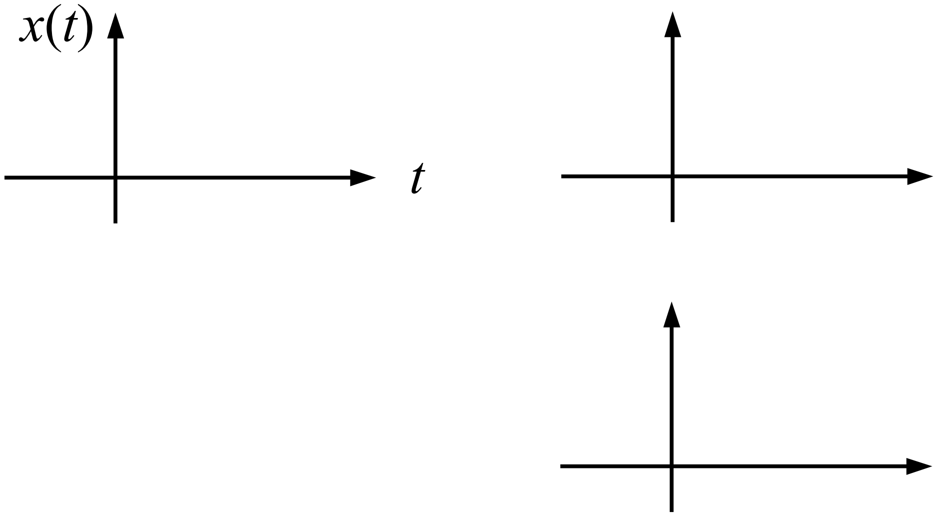
  $x(t) = 1, 0<t<1,$

  $\quad\;\; = 0$, ow

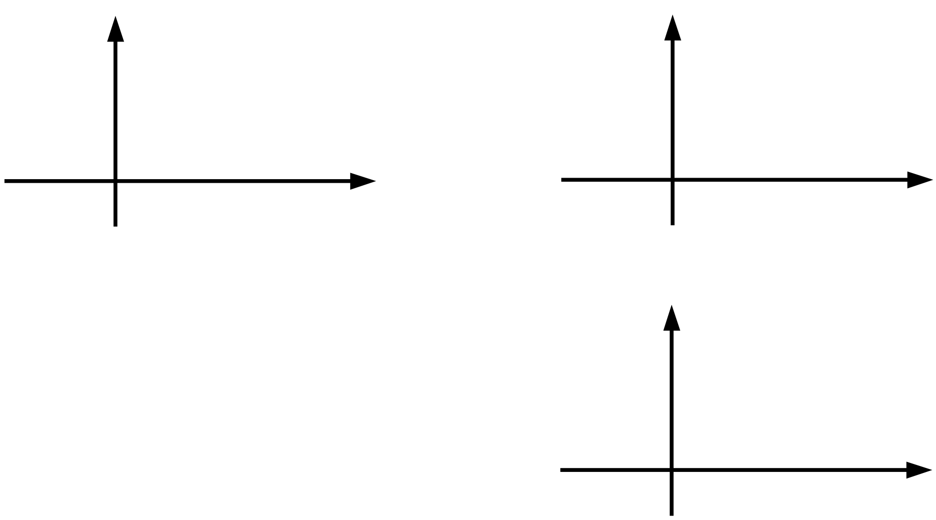
Check whether the above signals are energy or power signals

# 3) Transformation of Signals

- Time-shift




- Compression/expansion



- Plot $x(2t+2)$
  $x(2-t)$

## 4) Periodic/Aperiodic Signal

- Def: A periodic signal $n(t)$ is such that:

$$\exists \; T > 0 \;, \quad x(t) = x(t+T) \qquad \forall \; t$$

called period

- Fundamental period: Smallest value of $T$ for which above holds

- Example: $x(t) = \sin(\pi t)$

- $x[n] = \sin(\pi n/4)$

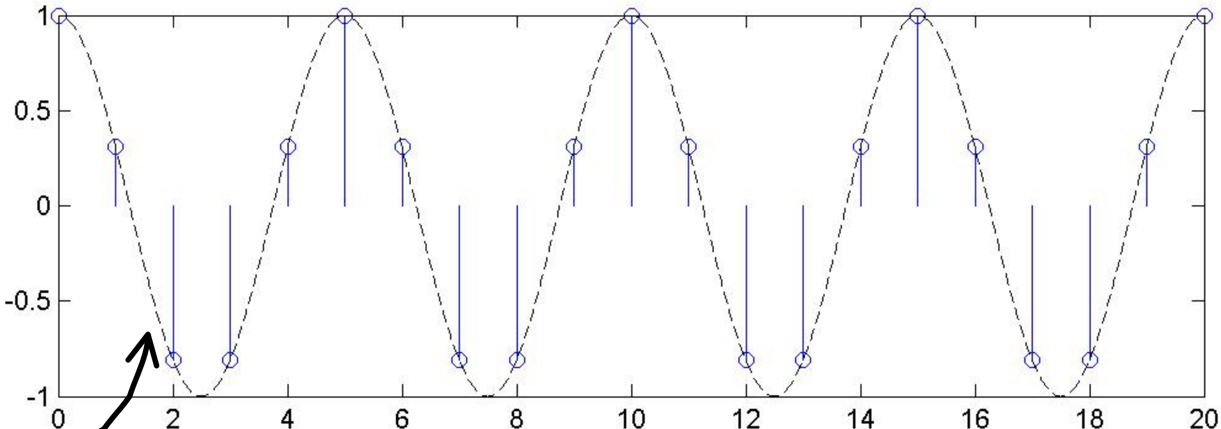
- $x_1(t) = 2 \sin(10t) - 6 \cos(6t)$

  $x_2(t) = 2 \sin(10\pi t) - 6 \cos(2t)$

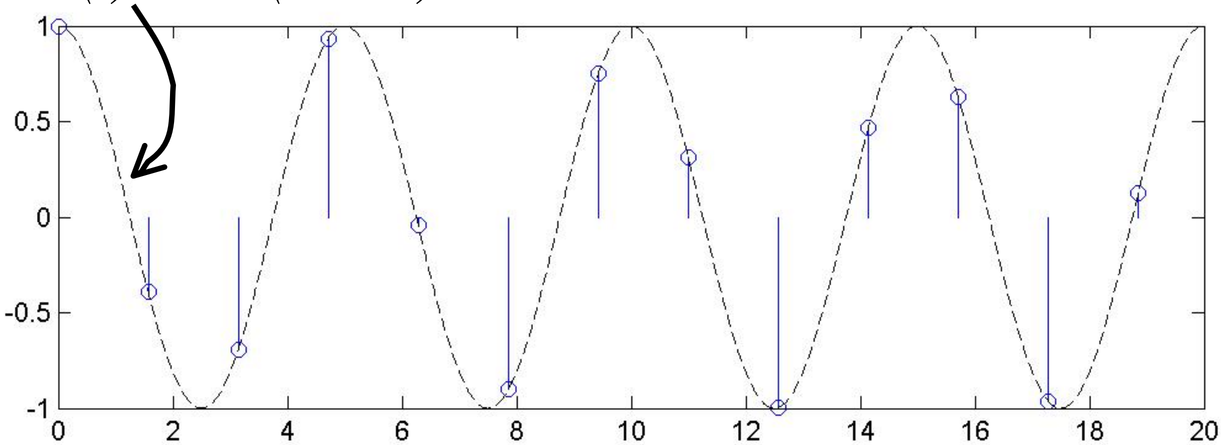
❖ Note: A continuous period signal may not be periodic in discrete form

Example:

$x_1[nT]=\cos[2\pi(nT)/5],\ T=1$




$x(t)=\cos(2\pi t/5)$




$x_2[nT]=\cos[2\pi(nT)/5],\ T=\pi/2$

## 5) Even/Odd Signals

- Def: A signal is said to be even if:

- Def: A signal is said to be odd if:

- Prop: All signals can be broken into even and odd parts:

  $E\ \{x(t)\} =$

  $O\ \{x(t)\} =$

- Prop: if $x_1(t)$ and $x_2(t)$ are even (odd) then:

  (1) $x_3(t) = x_1(t) + x_2(t)$ is even (odd)
  (2) $x_4(t) = x_1(t) \cdot x_2(t)$ is even

❖ Example:

$$x(t) = \begin{cases} A & 0 \le t \le 1 \\ 2A & 1 \le t \le 2 \\ 0 & ow(otherwise) \end{cases}$$

find $E\{x(t)\}$ ; $O\{x(t)\}$

# 6) Basic Types of Signals

- Real exponential signal

$$x(t) = Ae^{bt} \qquad b \in \mathbb{R}$$

- Complex exponential signal

$$x(t) = e^{st} \quad \text{with} \quad s = \sigma + j\omega_0$$

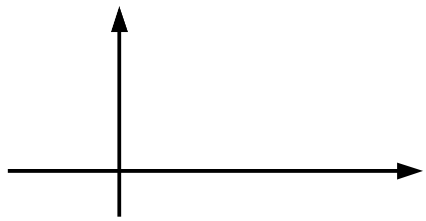
- Periodicity of a complex exponential

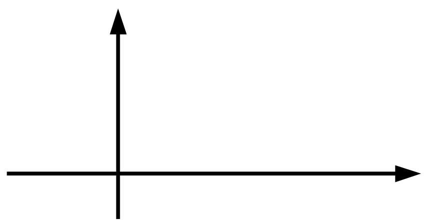
- Sinusoidal signal

$$x(t) = \underbrace{A}_{\text{amplitude}} \cos(\omega_0 t + \underbrace{\phi}_{\text{phase}})$$

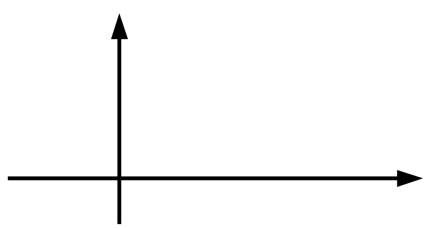
- Expressing complex exponential in terms of cosine and sine

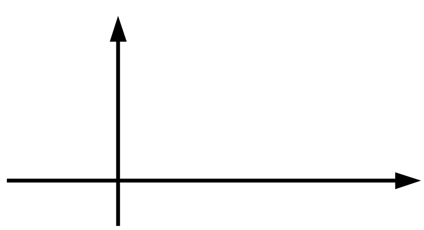
- Discrete-time unit impulse function





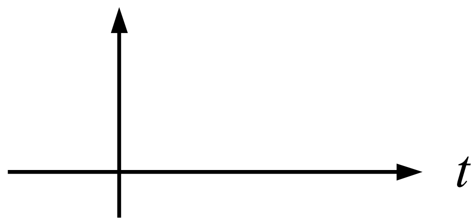
- Discrete-time step function



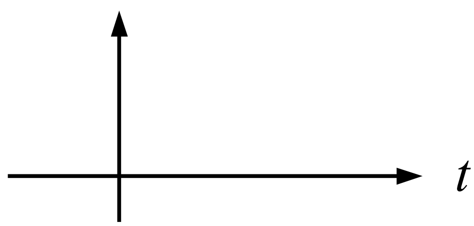


- Relationship between $\delta[n]$ and $u[n]$

- Continuous-time unit step function




- Continuous-time unit impulse function



- Properties of $\delta(t)$

- Some properties of the impulse function

  (1) Sifting property

(2) Scaling property

(3) Consequences

❖ Examples:

$$A = \int \delta(t)\cos(t)dt$$

$$B = \int e^{2t}\delta(t-4)dt$$

$$C = \int \delta(t)u(t)dt$$

$$D = \int x(t)\delta(-2t)dt$$

$$E = \int [e^{t}u(-t) + e^{-t}u(t)]dt$$

- Sinc function

$$\operatorname{sinc}(t) = \frac{\sin(\pi t)}{(\pi t)}$$