1-4 APPROXIMATE INTEGRATION TECHNIQUES AND APPLICATIONS

The salvage engineer may be required to calculate hydrostatic data for a casualty when curves of form or other documents are not available; for a casualty in an unusual condition, such as a ship floated upside down or on its side; or for portions of a ship that has been cut into sections. A ship's form consists of a number of intersecting surfaces, usually of nonmathematical form. Areas and volumes enclosed by these surfaces, as well as moments of areas and volumes, and second moments of area, must be determined to calculate hull hydrostatic characteristics.

For a curve plotted on an *xy* coordinate system, the area under the curve and moments, second moments (moments of inertia), and location of the centroid can be expressed as simple integrals. Since hull forms are seldom definable by mathematical equations, areas, moments, and volumes are calculated by manual integration methods rather than by direct integration. Manual integration methods are also used to evaluate any parameter that can be expressed as a curve of a function of some variable. For example, the total force, location of the center of effort, and force moment of an unevenly distributed force (such as current forces) can be determined from a curve showing the force distribution. Graphical and numerical manual integration methods are described in the following paragraphs.

1-4.1 Graphical Integration. An obvious way to calculate the area under a curve (or within a shape) is to plot the curve to scale on graph paper and count the squares under the curve. This method can be extended to calculate the first moment of area, $M_y = \int xy\, dx$, by multiplying the height (number of squares, y) in each column by its distance from the origin (x), and summing all such products. In the same way, the second moment is calculated by multiplying the height of each column by x^2. By adopting sign conventions and adjusting the location of the origin, moments can be calculated about any desired axis. Graphical integration of large, complex areas is very tedious, but can be very accurate for even the most complex or discontinuous curves.

1-4.2 Numerical Integration. Numerical integration methods, or *rules*, are based on the same premise as graphical integration; that the area under a curve can be closely approximated by breaking the area up into smaller shapes whose areas can be calculated or estimated easily, and summing the areas of these shapes. Most rules depend upon the substitution of a simple mathematical form for the actual curve to be integrated. The accuracy of the result depends upon the accuracy of the fit between the real and assumed curves.

1-4.3 Trapezoidal Rule. The trapezoidal rule substitutes a series of straight lines for a complex curve to allow integration of the curve in a simple tabular format. Conceptually, the trapezoidal rule is the simplest of the numerical integration rules.

A curvilinear shape can be approximated by a series of n trapezoids bounded by $n + 1$ equally spaced ordinates, y_0, y_1, y_2, y_3, ..., y_n, (at stations x_0, x_1, x_2, x_3, ..., x_n) as shown in Figure 1-5. If the station spacing is h, the area ($a_{0,1}$) of the first trapezoid is:

$$a_{0,1} = \frac{y_0 + y_1}{2}\, h$$

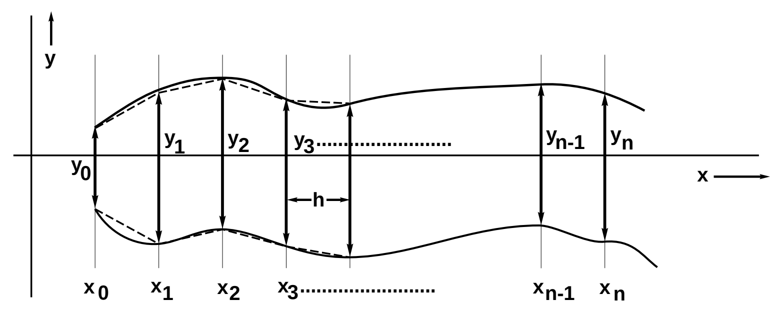


Figure 1-5. Curvilinear Figure Approximated by Series of Trapezoids.

The total area of the shape (A) is approximately equal to the sum of the areas of the trapezoids:

$$\begin{aligned} A &= a_{0,1} + a_{1,2} + a_{2,3} + \ldots + a_{n-1,n} \\ &= \frac{y_0 + y_1}{2}h + \frac{y_1 + y_2}{2}h + \frac{y_2 + y_3}{2}h + \ldots + \frac{y_{n-1} + y_n}{2}h \\ &= \frac{h}{2}\left(y_0 + 2y_1 + 2y_2 + 2y_3 \ldots + y_n\right) \\ &= h\left(\frac{y_0}{2} + y_1 + y_2 + y_3 + \ldots + \frac{y_n}{2}\right) \end{aligned}$$

This expression is called the *trapezoidal rule*, and can be used to calculate areas of any shape bounded by a continuous curve, simply by dividing the shape into a number of equal sections and substituting the ordinate values and the station spacing, or *common interval*, into the rule. The *common multiplier* for the trapezoidal rule is the common interval (h). If the common interval and common multiplier (CM) are separated into two factors, the common multiplier for the trapezoidal rule is 1.

The factors by which each ordinate is multiplied (½, 1, 1, 1, ..., ½) are the *individual multipliers* (m). The products of the individual multipliers and ordinates are called *functions of area*, $f(A)$. The area under the curve is thus expressed as:

$$A = \int y\, dx = h \Sigma f(A)$$

Because the trapezoidal rule substitutes a series of straight lines for the curve to be integrated, it is best suited for use with smooth, long-radius curves such as the waterlines of a ship. The rule underestimates the area under convex curves, and overestimates the area under concave curves. Accuracy increases as station spacing is decreased. If greater accuracy is required in regions of considerable curvature, e.g. at the ends of the ship, stations are taken at half-divisions. When half-spaced stations are used, the individual multipliers for the half-stations and adjacent stations must be adjusted. If, for example, a half-station is inserted between ordinates 1 and 2:

$$A = \frac{y_0 + y_1}{2}h + \frac{y_1 + y_{1.5}}{2}\frac{h}{2} + \frac{y_{1.5} + y_2}{2}\frac{h}{2} + \frac{y_2 + y_3}{2}h \; ... + \frac{y_{n-1} + y_n}{2}h$$

$$= \frac{h}{2}\left(y_0 + 1.5y_1 + y_{1.5} + 1.5y_2 + 2y_3 + ... + y_n\right)$$

$$= h\left(\frac{1}{2}y_0 + \frac{3}{4}y_1 + \frac{1}{2}y_{1.5} + \frac{3}{4}y_2 + y_3 + ... + \frac{1}{2}y_n\right)$$

The individual multiplier for the half-station is ½, and ¾ for the station on either side of it. A similar analysis will show that if several sequential half-stations are inserted (i.e., 2½, 3½, 4½, etc.) the multipliers for all stations and half-stations between the first and last half-stations is ½, and the multiplier for the two outlying whole stations is ¾. It may be more convenient to use the first form of the rule, to avoid divisors greater than 2, in which case all the individual multipliers are doubled.

1-4.4 Simpson's Rules. The replacement of a complex or small radius curve by a series of straight lines limits the accuracy of calculations, unless a large number of ordinates are used. Integration rules that replace the actual curve with a mathematical curve of higher order are more accurate. Simpson's rules assume that the actual curve can be replaced by a second-order curve (parabola). Figures 1-6 through 1-8 demonstrate the derivations of Simpson's rules.

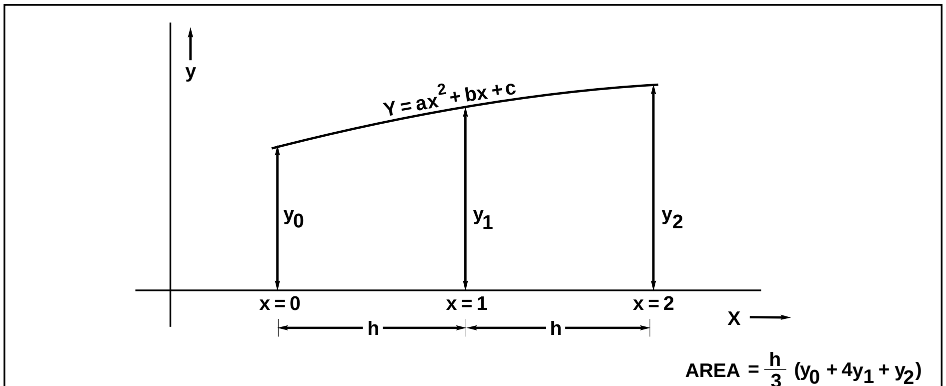


Figure 1-6. Simpson's Three-Ordinate Rule.

1-4.4.1 Simpson's First Rule. Figure 1-6 shows a curve of the form $y = ax^2 + bx + c$. It is expressed by three evenly spaced ordinates y_0, y_1 and y_2, at x = 0, 1, and 2 (station spacing = 1). The values of the ordinates are:

$$y_0 = a(0)^2 + b(0) + c = c \qquad \text{for } x = 1$$

$$y_1 = a(1)^2 + b(1) + c = a + b + c \qquad \text{for } x = 1$$

$$y_2 = a(2)^2 + b(2) + c = 4a + 2b + c \qquad \text{for } x = 2$$

The area under the curve is:

$$A = \int_0^2 (ax^2 + bx + c)dx = \frac{ax^3}{3} + \frac{bx^2}{2} + cx\Big|_0^2 = \frac{8}{3}a + 2b + 2c$$

Now $c = y_0$ and $y_1 = y_0 + a + b$, and $y_2 = y_0 + 4a + 2b$. Substituting and solving for a and b:

$$y_2 - 2y_1 = y_0 + 2b + 4a - 2y_0 - 2b - 2a = -y_0 + 2a$$

$$\therefore \; a = \frac{(y_2 - 2y_1 + y_0)}{2}$$

$$b = y_1 - y_0 - a = y_1 - y_0 - \frac{(y_2 - 2y_1 + y_0)}{2} = -\frac{3}{2}y_0 - \frac{y_2}{2} + 2y_1$$

Area (A) is expressed as:

$$A = \frac{8}{3}a + 2b + 2c = \frac{8}{3}\left(\frac{y_2 - 2y_1 + y_0}{2}\right) + 2\left(-\frac{3}{2}y_0 - \frac{y_2}{2} + 2y_1\right) + 2y_0$$

$$= 2y_0 - 3y_0 - y_2 + 4y_1 + \frac{4}{3}y_2 - \frac{8}{3}y_1 + \frac{4}{3}y_0 = \frac{1}{3}y_0 + \frac{4}{3}y_1 + \frac{1}{3}y_2$$

$$= \frac{1}{3}\left(y_0 + 4y_1 + y_2\right)$$

For an ordinate spacing of h rather than unity:

$$A = \frac{h}{3}(y_0 + 4y_1 + y_2)$$

This relationship is *Simpson's first rule*, or *3-ordinate rule*, commonly called *Simpson's rule*. The rule calculates correctly the area under a second order curve and will approximate the area under any curve that passes through the same three points. The accuracy depends on how closely the actual curve approaches the parabolic form assumed by the rule. Simpson's Rule is the numerical integration rule used most widely for ship calculations.

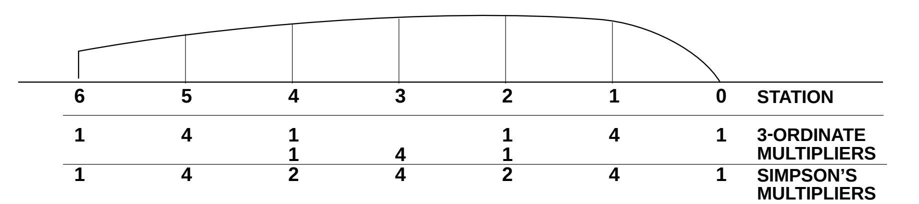


Figure 1-7. Simpson's Multipliers for Long Curve.

The rule can be extended to calculate the area under a long nonparabolic curve such as a ship's waterline. If the length of the curve is divided into enough equal parts, as shown in Figure 1-7, it can be reasonably approximated by a series of parabolic segments. For a curve divided into n equal parts, the area between the first (0) and third (2) ordinates would be given by:

$$A_{0-2} = \frac{h}{3}\,(y_0 + 4y_1 + y_2)$$

where:

$A_{0\text{-}2}$	=	area under the curve between the first and third ordinates
h	=	distance between ordinates = L/n
L	=	length of the curve
n	=	number of sections between ordinates = number of ordinates - 1

Similarly, the area between the third (2) and fifth (4) ordinates would be:

$$A_{2-4} = \frac{h}{3}\,(y_2 + 4y_3 + y_4)$$

The area between the fifth (4) and seventh (6) ordinates:

$$A_{4-6} = \frac{h}{3}\,(y_4 + 4y_5 + y_6)$$

and so on.

The total area is the sum of all the two section areas:

$$A = A_{0-2} + A_{2-4} + A_{4-6} + \ldots + A_{n-2-n}$$

$$= \frac{h}{3}\left(y_0 + 4y_1 + 2y_2 + 4y_3 + 2y_4 + 4y_5 + 2y_6 + \ldots + y_n\right)$$

This is the general form of Simpson's rule. Since the rule consists of a summation of areas over two sections of a curve divided into a number of equal sections, the curve must be divided into an *even* number of sections (by an *odd* number of stations) to apply the rule. The *common multiplier* (*CM*) is ⅓; the individual multipliers are 1, 4, 2, 4, 2, 4,..., 2, 4, 1. The derivation of the individual multipliers as a tabular summation of the 3-ordinate rule multipliers for each two adjacent sections is shown in Figure 1-7.

In regions where the slope of the curve changes rapidly, the accuracy of the rule can be increased by inserting intermediate (half-spaced) stations. When half-spaced stations are used, the individual multipliers are modified. For example, a half-station could be inserted at 2½ were there a rapid change in form between the third and fourth stations of the curve in Figure 1-7. The area between the first and second stations is calculated as before:

$$A_{0-2} = \frac{h}{3}\,(y_0 + 4y_1 + y_2)$$

With the insertion of the half-station (2½), the 3-ordinate rule can be applied to the area between the third and fourth ordinates ($A_{2\text{-}3}$), with an ordinate spacing of $h/2$:

$$A_{2-3} = \frac{\frac{h}{2}}{3}\left(y_2 + 4y_{2.5} + y_3\right) = \frac{h}{3}\left(\frac{y_2}{2} + 2y_{2.5} + \frac{y_3}{2}\right)$$

The area between the fourth and sixth stations ($A_{3\text{-}4}$) is now:

$$A_{3-4} = \frac{h}{3}\,(y_3 + 4y_4 + y_5)$$

and so on. The total area is:

$$\begin{aligned} A &= A_{0-2} + A_{2-3} + A_{3-5} + \ldots + A_{n-1-n} \\ &= \frac{h}{3}\left(y_0 + 4y_1 + y_2 + \frac{y_2}{2} + 2y_{2.5} + y + \frac{2y_3}{2} + y_3 + 4y_4 + y_5 + \ldots + y_n\right) \\ &= \frac{h}{3}\left(y_0 + 4y_1 + 1\frac{1}{2}y_2 + 2y_{2.5} + 1\frac{1}{2}y_3 + 4y_4 + 2y_5 + \ldots + y_n\right) \end{aligned}$$

Note that unless another half-spaced station is inserted, the number of sections (n) will be even, and the rule unworkable. Intermediate stations can be inserted at any equal division of the station spacing (third-stations, quarter-stations, etc.) and multipliers deduced in a similar manner. Intermediate stations can be inserted anywhere along the length of the curve so long as two rules are followed:

- An even number of intermediate stations must be inserted, so that the total number of segments remains even (total number of ordinates is odd).
- Intermediate stations must be inserted so there are an even number of segments in each group of consecutive whole or partial segments (each group of whole or partial segments includes an odd number of ordinates).

Intermediate stations are commonly used near the ends of waterlines where the hull form changes rapidly with respect to length. The individual multipliers can be quickly determined by tabulating and summing the appropriate 3-ordinate rule multipliers as shown in Figure 1-8.

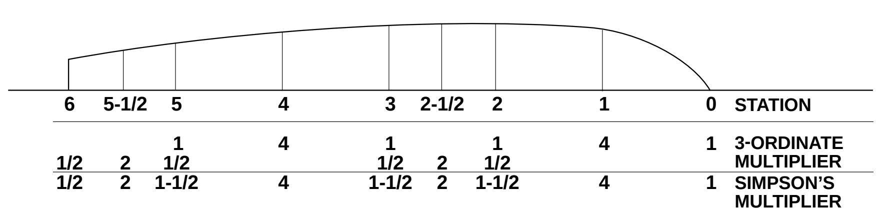


Figure 1-8. Simpson's Multipliers with Half-Spaced Stations.

1-4.4.2 Simpson's Second Rule. Rules can be deduced, in a similar manner, for areas bounded by different numbers of evenly spaced ordinates, or by unevenly spaced ordinates. For four evenly spaced ordinates:

$$A = \frac{3h}{8}\,(y_0 + 3y_1 + 3y_2 + y_3)$$

This is *Simpson's second* or *three-eighths Rule*. The general form is:

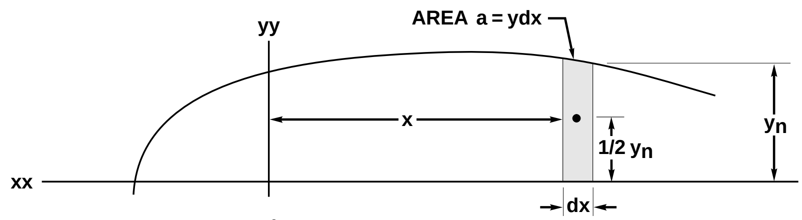
$$A = \frac{3h}{8}\,(y_0 + 3y_1 + 3y_2 + 2y_3 + 3y_4 + 3y_5 + 2y_6 + \ldots + y_n)$$

Simpson's second rule can be used with 4 + 3i ordinates, where i is a positive integer (i.e., 4, 7, 10, 13, etc.).

1-4.5 Applications. The derivations of Simpson's rules and the trapezoidal rule were demonstrated with area computations to aid conceptualization, but the rules can integrate any function that can be plotted on Cartesian coordinates. If, for example, the ordinates represent sectional areas along a ship's length for a given waterline, the products of the multipliers and ordinates are functions of volume, $f(V)$, and their summation (integral of the curve) is the volume of displacement. Calculation of areas, moments, centroids, and second moments of areas by the are described in the following paragraphs.

1-4.5.1 Moments and Centroids. As shown in Figure 1-9, the moment of an elemental strip of area about some vertical axis *YY* is *xydx*. To determine the moment of a larger area about the axis, the integral $M = \int xy\,dx$ must be evaluated. Instead of multiplying the value of y at each station by the appropriate multiplier, the value xy is multiplied, where x is the distance from the station to the reference axis, and dx is the width of each strip, or the common interval h. The value $y\,dx = hy_n$ is the area of the strip a_n; the first moment of this area about some reference axis *YY* is:

$$M_{YY} = x_n h y_n = x_n a_n$$



FOR SHADED STRIP: $a = ydx$

$$i = \frac{ay^2}{12} = \frac{y^3dx}{12}$$

$$M_{yy} = xa = x(ydx)$$

$$I_{yy} = x^2a = x^2(ydx)$$

$$I_{xx} = \frac{y^2}{2}a + i = \frac{y^2}{2}(ydx) + \frac{y^3dx}{12}$$

Figure 1-9. Variables for Moment and Second-Moment Calculations.

The total moment is the sum of the moments of all the strips, that is, the integral of the incremental moments along the length:

$$M_{YY} = \int_0^L x_n a_n\,dx$$

The integral can be evaluated numerically:

$$\int x_n a_n\,dx = \Sigma x_n CMf(A) = CM\Sigma x_n f(A)$$

where:

CM	=	common multiplier for the appropriate integration rule
$f(A)$	=	function of area = $m_n y_n$
m_n	=	common multiplier for the appropriate rule and station

If the reference axis is chosen to fall on an ordinate station, then the moment arms have the common interval (h) as a common factor, i.e., $x_n = s_n h$, where x_n is the moment arm and s_n is the number of stations from the reference axis to station n. The factor h can be brought outside the summation:

$$M_{YY} = CMh\,\Sigma\,s_n f(A)$$

The products of the number of stations from the reference axis and the functions of area, $s_n f(A)$, are the functions of moment $f(M)$:

$$M_{YY} = CMh\,\Sigma\,f(M)$$

The distance from the centroid of the shape to the reference axis (x') is the moment divided by the area:

$$x' = \frac{M_{YY}}{A} = \frac{CMh\Sigma f(M)}{CM\Sigma f(A)} = \frac{\Sigma f(M)}{\Sigma f(A)}h$$

The centroid of a symmetrical shape lies on the axis of symmetry, and its location can be defined by summing moments about a single axis perpendicular to the axis of symmetry. To precisely locate the centroid of an asymmetrical shape, moments must be summed about another, perpendicular, axis. The calculation can be performed by taking ordinates perpendicular to the first set and integrating with respect to *y* rather than *x*. Moments about an axis *XX* can also be determined using y ordinates, but with slightly less accuracy. Referring again to Figure 1-9, the moment about axis *XX* of the elemental strip *dx* is:

$$M_{XX} = \left(\frac{y}{2}\right)a = \left(\frac{y}{2}\right)y\,dx = \left(\frac{y^2}{2}\right)dx$$

where *y* is the height of the strip, and *a* its area. The total moment is the integral of the incremental moments along the length, and the integral can be evaluated numerically:

$$M_{XX} = \frac{\int_0^L y_n}{2}\, a_n dx = \frac{\Sigma y_n}{2}\, CMf(A)_n = \frac{CM\Sigma y_n f(A)_n}{2}$$

The product of the *y* ordinate and the function of area for each segment can be defined as the function of moment about *x*, $f(M_{XX})$:

$$f\left(M_{XX}\right) = yf(A) = y^2 m_n$$

$$M_{XX} = \frac{CM}{2}\,\Sigma\, f\left(M_{XX}\right)$$

where m_n is the individual multiplier for the *n*th ordinate. The distance from the centroid of the shape to the axis *XX* (y') is the moment divided by the area:

$$y' = \frac{M_{XX}}{A} = \frac{\frac{CM}{2}\Sigma f\left(M_{XX}\right)}{CM\Sigma f(A)} = \frac{\Sigma f\left(M_{XX}\right)}{2\Sigma f(A)}$$

Moments can be summed about any axis, although it is simplest to sum them about an axis through x_0 so that the number of stations from the reference axis is simply the station number. For ship calculations, moments are often summed about the midships section to reduce the size of the products and sums for manual calculation, and because the centers of flotation, buoyancy, and gravity normally lie near midships. When moments are summed about a station other than an end station, a sign convention must be adopted so that distances to one side of the reference axis (and therefore moments and functions of moments) are negative.

1-4.5.2 Second Moments of Area. The second moment of area (moment of inertia, *I*) of a plane shape about an axis *YY* parallel to the vertical ordinates is given by:

$$I_{YY} = \int_0^L x^2 y\, dx$$

where:

I_{YY}	=	second moment of area about some axis *YY*
x	=	distance from axis *YY* to elemental vertical strip of height *y* and width *dx*
L	=	length of the area whose second moment is desired, measured along an axis perpendicular to *YY*

An analysis similar to that taken for the calculation of first moments will show that the second moment of the area under a curve is calculated by:

$$I_{YY} = CMh^2\, \Sigma\, f(I_{YY})$$

where:

CM	=	common multiplier
h	=	common interval
$f(I_{YY})$	=	function of second moment about axis $YY = s_n^2 m_n y_n$
s_n	=	number of stations from axis *YY* to station *n*
m_n	=	individual multiplier for station *n*
y_n	=	height of the ordinate at station *n*

The second moment of an area (moment of inertia) is always smallest about an axis through its centroid, (the *neutral axis* in bending stress analysis). If moment of inertia about some axis *YY*, parallel to the neutral axis is known, the moment of inertia about the neutral axis (I_{NA}) is found by the parallel axis theorem:

$$I_{NA} = I_{YY} - Ad^2$$

where d is the distance from axis *YY* to the neutral axis, and A is the total area of the section.

The second moment of area about an axis *XX* perpendicular to axis *YY* can be calculated by taking ordinates perpendicular to the first set and integrating twice with respect to y rather than x. To determine the second moment about a horizontal axis of symmetry, such as the moment of inertia of a waterplane about its centerline, the integration can also be performed using the original set of ordinates. In Figure 1-9 (Page 1-20), y is the half-ordinate of an incremental strip of a waterplane measured from the centerline. The second moment of area of the incremental strip about the centerline is:

$$i_{xx} = \left(\frac{y}{2}\right)^2 a + i_0 = y\,dx\left(\frac{y}{2}\right)^2 + \left(\frac{1}{12}\right)y^3dx = \left(\frac{1}{3}\right)y^3dx$$

where:

- i_{xx} = second moment of area of incremental strip about the centerline
- a = area of the incremental strip
- i_0 = second moment of area of the incremental strip about a horizontal centroidal axis
  = $(1/12)y^3dx$ if strip is assumed to be rectangular
- dx = width of the incremental strip

The total second moment of half-waterplane area is:

$$I_{XX,\ \mathrm{half}} = \int_0^L \left(\frac{1}{3}\right)y^3dx = \left(\frac{1}{3}\right)\int_0^L y^3dx$$

The second moment of the total area is twice this amount, and this will be the second moment about the centerline, since the waterplane is symmetrical about the centerline. The integration $\int y^3dx$ can be performed numerically:

$$I_{XX} = 2\left(\frac{CM\,h}{3}\right)\Sigma\, f\,(I_{XX})$$

where:

- CM = common multiplier
- h = common interval
- $f(I_{XX})$ = function of second moment about axis XX = $m_n y_n^3$
- m_n = individual multiplier for station n
- y_n = height of the half-ordinate at station n

1-4.5.3 Volumes and Centroids of Volume. Volumes are calculated by integrating a curve of sectional areas. To calculate the volume of the tank shown in Figure 1-10, the shape is first *cut* at several stations to form section outlines. The area of each section is calculated, and the areas taken as ordinates along the length of the tank. Integrating the area ordinates by the trapezoidal rule:

$$V = \int a\ dx = h\ \Sigma f(V)$$

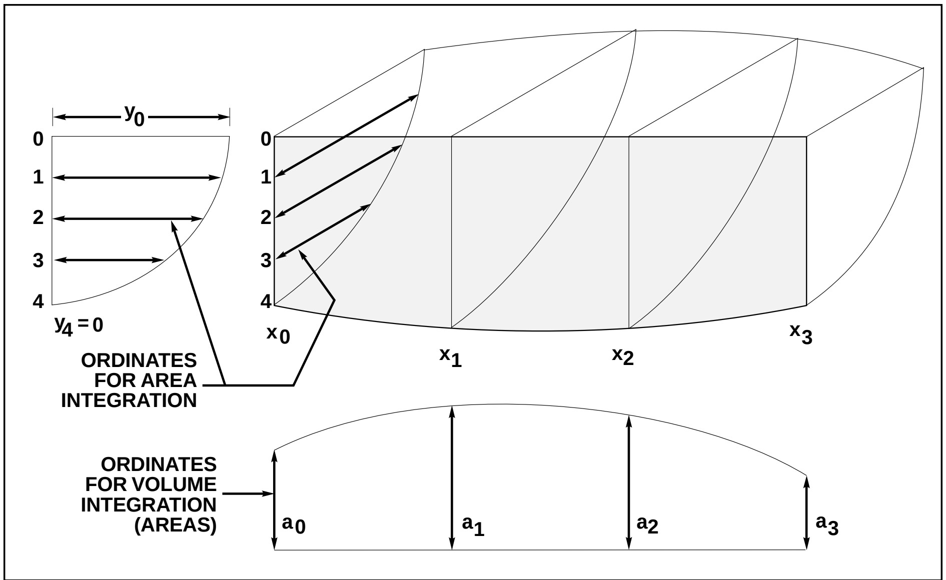


Figure 1-10. Determination of Volume by Numerical Integration.

where:

- $f(V)$ = function of volume = $m_n a_n$
- m_n = individual multiplier for station n
- a_n = area of section at station n

The moment of volume about some axis *YY* is:

$$M_{YY} = h^2 \sum f(M)$$

where:

$f(M)$ = function of moment of volume about axis $YY = s_n m_n a_n$
s_n = number of stations from axis YY to station n

The distance of the centroid from axis *YY*:

$$d = \frac{h^2 \Sigma f(M)}{h \Sigma f(V)} = \frac{\Sigma f(M)}{\Sigma f(V)} h$$

These forms are exactly the same as those used to calculate areas and moments and centroids of areas; the only difference is that ordinate values represent areas rather than linear distances. Integrations can be performed along additional axes to precisely locate the centroid of the shape.

1-4.5.4 General Forms for Area and Moment Calculations. Calculation of areas, moments, centroids, and second moments of area by Simpson's first and second rules can be expressed in general forms:

$$A = (CM) h \Sigma f(A)$$

$$M_{YY} = (CM) h \Sigma f(M)$$

$$M_{XX} = \left(\frac{CM}{2}\right) \Sigma f\left(M_{XX}\right)$$

$$x' = \frac{(CM) h \Sigma f(M)}{(CM) \Sigma f(A)} = \frac{\Sigma f(M)}{\Sigma f(A)} h$$

where:

A	=	area under a curve between selected stations		
M_{YY}	=	first moment of area about axis YY		
M_{XX}	=	first moment of area about axis XX		
x'	=	distance from centroid of area to axis YY		
y'	=	distance from centroid of area to axis XX		
I_{YY}	=	second moment of area about axis YY		
I_{XX}	=	second moment of area about centerline axis XX		
CM	=	common multiplier for the appropriate rule (1, 1/3, 3/8, etc)		
h	=	common interval		
$f(A)$	=	function of area	=	$m_n y_n$
$f(M)$	=	function of moment about YY	=	$s_n m_n y_n = s_n f(A)$
$f(M_{XX})$	=	function of moment about XX	=	$m_n y_n^2 = y_n f(A)$
$f(I_{YY})$	=	function of second moment about YY	=	$s_n^2 m_n y_n = s_n f(M) = s_n^2 f(A)$
$f(I_{XX})$	=	function of second moment about XX	=	$m_n y_n^3$
s	=	number of stations from axis YY (or integration start point) to station n		
m	=	individual multiplier for station n for the appropriate rule		
y_n	=	height of the ordinate at station n (half-ordinate for I_{XX})		

Examples 1-1 and 1-2 demonstrate the use of the trapezoidal rule and Simpson's rule to calculate waterplane functions for an FFG-7 Class ship.

EXAMPLE 1-1

CALCULATION OF WATERPLANE PROPERTIES BY TRAPEZOIDAL RULE

Using 11- and 21-ordinate trapezoidal rules, calculate the waterplane area (A_{WP}), location of the center of flotation (*LCF*), moment of inertia of the waterplane about the centerline (I_{CL}) and a transverse axis through the *LCF* (I_{CF}), tons per inch immersion in saltwater (*TPI*), and waterplane coefficient (C_{WP}) for the 16-foot waterline of an FFG-7 Class ship. Compare these values with actual data.

Actual Properties:

L	= 408 ft	I_{CF}	= 135,888,480 ft^4
B_{max}	= 45.6 ft	I_{CL}	= 1,664,145 ft^4
A_{WP}	= 13,860 ft^2	*TPI*	= 33 tons/in
LCF	= 24.1 ft aft of midships = 228.1 ft from forward perpendicular	C_{WP}	= 0.745

Since the waterplane is symmetrical about its centerline, areas and moments can be found by integrating one side of the waterplane along the centerline with *half-ordinates (halfbreadths)* measured from the centerline, and doubling the results. Halfbreadths for the 16-foot waterline, in feet, inches, and eighths, are taken from Figure FO-1. The integrations are best performed in a tabular format. To integrate on 11 ordinates, halfbreadths for stations 0, 2, 4, 6, 8, 10, 12, 14, 16, 18, and 20 are used.

Integration on 11 ordinates:

Station	Ordinate, y		Multiplier m	$f(A)$ $m \times y$	Lever s	$f(M)$ $s \times f(A)$	$f(I_{YY})$ $s \times f(M)$	$f(I_{XX})$ $m \times y^3$
	ft-in-1/8	ft		ft^2	ft	ft^3	ft^4	ft^4
0	0 - 4 - 5	0.39	½	0.19	0	0.0	0.0	0.03
2	6 -10 - 5	6.89	1	6.89	1	6.89	6.89	327.1
4	12-11 - 0	12.92	1	12.92	2	25.84	51.68	2156.7
6	17- 9 - 2	17.77	1	17.77	3	53.31	159.93	5611.3
8	20-11 - 5	20.97	1	20.97	4	83.88	335.52	9221.4
10	22- 7 - 1	22.59	1	22.59	5	112.95	564.75	11527.9
12	22- 8 - 3	22.70	1	22.70	6	136.20	817.20	11697.1
14	21- 8 - 4	21.71	1	21.71	7	151.97	1063.37	10232.4
16	19- 7 - 1	19.59	1	19.59	8	156.72	1253.76	7518.0
18	16- 8 - 6	16.73	1	16.73	9	150.57	1355.13	4682.6
20	12- 7 - 0	12.58	½	6.29	10	62.90	629.00	995.4
				168.34		941.23	6237.65	63969.9

$h = 408/10 = 40.8$ ft

$A_{WP} = 2h\sum f(A) = 2(40.8)(168.34) = 13{,}736.5$ ft^2

$M_{FP} = 2h^2 \sum f(M) = 2(40.8)^2(941.23) = 3{,}133{,}618$ ft^3

$$x' = \frac{\sum f(M)}{\sum f(A)}\,h = \frac{941.23}{168.34}(40.8) = 228.1 \text{ ft from } FP = LCF$$

$I_{FP} = 2h^3 \sum f(I_{YY}) = 2(40.8)^3(6237.65) = 847{,}288{,}842$ ft^4

$I_{CF} = I_{FP} - Ad^2 = 847{,}288{,}842 - 13{,}736.5(228.1)^2 = 132{,}516{,}043$ ft^4

$I_{CL} = 2(h/3) \sum f(I_{XX}) = 2(40.8/3)(63{,}969.9) = 1{,}739{,}981$ ft^4

$TPI = A_{WP}/420 = 13{,}736.5/420 = 32.7$ tons

$C_{WP} = A_{WP}/(LB) = 13{,}736.5/(408 \times 45.6) = 0.738$

Integration on 21 ordinates:

Station	Ordinate, y		Multiplier m	$f(A)$ s	Lever	$f(M)$	$f(I_{YY})$	$f(I_{XX})$
	ft-in-1/8	ft		ft^2	ft	ft^3	ft^4	ft^4
0	0 - 4 - 5	0.39	1/2	0.19	0	0.0	0.0	0.03
1	3 - 7 - 6	3.65	1	3.65	1	3.65	3.65	48.6
2	6 -10 - 5	6.89	1	6.89	2	13.78	27.56	327.1
3	10- 0 - 2	10.02	1	10.02	3	30.06	90.18	1006.0
4	12-11 - 0	12.92	1	12.92	4	51.68	206.72	2156.7
5	15- 6 - 1	15.51	1	15.51	5	77.55	387.75	3731.1
6	17- 9 - 2	17.77	1	17.77	6	106.62	639.72	5611.3
7	19- 6 - 7	19.57	1	19.57	7	136.99	958.93	7495.0
8	20-11 - 5	20.97	1	20.97	8	167.76	1342.08	9221.4
9	21-11 - 5	21.97	1	21.97	9	197.73	1779.57	10604.5
10	22- 7 - 1	22.59	1	22.59	10	225.90	2259.00	11527.9
11	22- 9 - 4	22.79	1	22.79	11	250.69	2757.59	11836.8
12	22- 8 - 3	22.70	1	22.70	12	272.40	3268.80	11697.1
13	22- 3 - 7	22.32	1	22.32	13	290.16	3772.08	11119.4
14	21- 8 - 4	21.71	1	21.71	14	303.94	4255.16	10232.4
15	20- 9 - 5	20.80	1	20.80	15	312.00	4680.00	8998.9
16	19- 7 - 1	19.59	1	19.59	16	313.44	5015.04	7518.0
17	18- 2 - 1	18.18	1	18.18	17	309.06	5254.02	6008.7
18	16- 8 - 6	16.73	1	16.73	18	301.14	5420.52	4682.6
19	15- 1 - 0	15.01	1	15.01	19	285.19	5418.61	3381.8
20	12- 7 - 0	12.58	1/2	6.29	20	125.80	2516.00	995.4
				338.18		3775.54	50052.98	128200.7

$h = 408/20 = 20.4$ ft

$A_{WP} = 2h\sum f(A) = 2(20.4)(338.18) = 13{,}797.5$ ft^2

$M_{FP} = 2h^2 \sum f(M) = 2(20.4)^2(3775.54) = 3{,}142{,}457$ ft^3

$$x' = \frac{\sum f(M)}{\sum f(A)}\,h = \frac{3775.54}{338.18}(20.4) = 227.8 \text{ ft from } FP = LCF$$

$I_{FP} = 2h^3 \sum f(I_{YY}) = 2(20.4)^3(50{,}052.98) = 849{,}865{,}964$ ft^4

$I_{CF} = I_{FP} - Ad^2 = 849{,}865{,}964 - 13{,}797.6(227.8)^2 = 134{,}155{,}856$ ft^4

$I_{CL} = 2(h/3) \sum f(I_{XX}) = 2(20.4/3)(128{,}200.7) = 1{,}743{,}529$ ft^4

$TPI = A_{WP}/420 = 13{,}797.6/420 = 32.9$ tons

$C_{WP} = A_{WP}/(LB) = 13{,}797.6/(408 \times 45.6) = 0.742$

Comparison:

	Actual	11 Ordinate		21 Ordinate	
		Value	Error, %	Value	Error, %
A_{WP}, ft^2	13,860.0	13,737.8	0.88	13,797.500	0.45
LCF, ft fm FP	228.1	228.1	0.00	227.800	0.13
I_{CF}, ft^4	135,888,480	132,502,924	2.49	134,155,856.000	1.28
I_{CL}, ft^4	1,664,145	1,739,981	4.56	1,743,529.000	4.77
TPI, tons/in	33	32.7	0.91	32.900	0.30
C_{WP}	0.745	0.738	0.94	0.742	0.40

EXAMPLE 1-2

CALCULATION OF WATERPLANE PROPERTIES BY SIMPSON'S RULE

Use Simpson's first rule with 11 ordinates to calculate the waterplane properties that were calculated in Example 1-1. Compare the results with actual data and the results by trapezoidal rule.

Ship dimensions and actual waterplane properties are the same as for Example 1-1. Halfbreadths for stations 0, 2, 4, 6, 8, 10, 12, 14, 16, 18, and 20 from Figure FO-1 are used to integrate on 11 stations. Integration:

Station	Ordinate, y		Multiplier m	$f(A7)$ $m \times y$	Lever s	$f(M)$ $s \times f(A)$	$f(I_{YY})$ $s \times f(M)$	$f(I_{XX})$ $m \times y^3$
	ft-in-1/8	ft		ft^2	ft	ft	ft^4	ft^4
0	0 - 4 - 5	0.39	1	0.39	0	0.0	0.0	0.06
2	6 -10 - 5	6.89	4	27.56	1	27.56	27.56	1308.3
4	12-11 - 0	12.92	2	25.84	2	51.68	103.36	4313.4
6	17- 9 - 2	17.77	4	71.08	3	213.24	639.72	22445.1
8	20-11 - 5	20.97	2	41.94	4	167.76	671.04	18442.7
10	22- 7 - 1	22.59	4	90.36	5	451.80	2259.00	46111.4
12	22- 8 - 3	22.70	2	45.40	6	272.40	1634.40	23394.2
14	21- 8 - 4	21.71	4	86.84	7	607.88	4255.16	40929.8
16	19- 7 - 1	19.59	2	39.18	8	313.44	2507.52	15036.0
18	16- 8 - 6	16.73	4	66.92	9	602.28	5420.52	18730.4
20	12- 7 - 0	12.58	1	12.58	10	125.80	1258.00	1990.9
				508.09		2,833.84	18,776.28	192,702.4

$h = 408/10 = 40.8$ ft

$A_{WP} = \frac{2}{3}\, h \sum f(A) = \frac{2}{3}(40.8)(508.09) = 13{,}820.1\ ft^2$

$M_{FP} = \frac{2}{3}\, h^2 \sum f(M) = \frac{2}{3}(40.8)^2(2833.84) = 3{,}144{,}882\ ft^3$

$$x' = \frac{\sum f(M)}{\sum f(A)}\, h = \frac{2833.84}{508.09}(40.8) = 227.6 \text{ ft from FP} = LCF$$

$I_{FP} = \frac{2}{3}\, h^3\, f(I_{YY}) = \frac{2}{3}(40.8)^3(18{,}776.28) = 850{,}156{,}311\ ft^4$

$I_{CF} = I_{FP} - Ad^2 = 850{,}156{,}311 - 13{,}820.1(227.6)^2 = 134{,}508{,}685\ ft^4$

$I_{CL} = \frac{2}{3}(h/3) \sum f(I_{XX}) = \frac{2}{3}(40.8/3)(192{,}702.4) = 1{,}747{,}168\ ft^4$

$TPI = A_{WP}/420 = 13{,}820.1/420 = 32.9$ tons

$C_{WP} = A_{WP}/(LB) = 13{,}820.1/(408 \times 45.6) = 0.743$

Comparison:

	Actual Value	11 Ordinate Simpson's Rule		Trapezoidal Rule Error, %	
		Value	Error, %	11 Ordinate	21 Ordinate
A_{WP}, ft^2	13,860	13,820.1	0.29	0.88	0.45
LCF, ft fm FP	228.1	227.6	0.22	0.00	0.13
I_{CF}, ft^4	135,888,480	134,508,685	1.02	2.49	1.28
I_{CL}, ft^4	1,664,145	1,747,168	4.99	4.56	4.77
TPI, tons/in	33	32.9	0.30	0.91	0.30
C_{WP}	0.745	0.743	0.27	0.92	0.40

The accuracy of an 11-ordinate Simpson's rule compares favorably with that of a 21-ordinate trapezoidal rule. Simpson's rule with 21 ordinates is only marginally more accurate than with 11 ordinates for this waterplane shape. Note that Simpson's rule calculates the moment of inertia about the centerline with slightly less accuracy than the trapezoidal rule. The derivation of the form: $I_{CL} = (CM)(h/3) \sum f(I_{XX})$ assumes a constant ordinate over the entire section (see Paragraph 1-4.3.3). The Simpson's multipliers do not correct for this assumption. The constant-ordinate assumption is essentially correct for very full ships and barges with extensive parallel midbody, and will yield very accurate values for I_{CL}. Accuracy of I_{CL} calculations for fine-lined ships can be increased only by using very close station spacing or integrating along an axis perpendicular to the centerline. The ± 5 percent accuracy shown here should be sufficiently accurate for most salvage work.

1-4.6 Other Simpson's Rule Forms. Simpson's rules can be derived for numbers of ordinates for which the first two rules do not apply, and to determine areas of "left over" segments at the ends of curves.

1-4.6.1 5, 8, Minus One and 3, 10, Minus One Rules. An additional Simpson's rule, known as the *5, 8, minus one rule*, is used to determine the area between two ordinates when three consecutive ordinates are known. For ordinates y_0, y_1, and y_2, the area between the first and second ordinates is given by:

$$A_{0\text{-}1} = \frac{1}{12} h(5y_0 + 8y_1 - y_2)$$

The area between the second and third ordinates can be found by applying the rule backwards:

$$A_{1\text{-}2} = \frac{1}{12} h(-y_0 + 8y_1 + 5y_2)$$

The validity of the 5, 8, minus one rule can be verified by observing that the sum of the expressions for the two sectional areas is the 3-ordinate rule:

$$\begin{aligned} A &= A_{0-1} + A_{1-2} = \frac{1}{12} h\left[(5y_0 + 8y_1 - y_2) + (-y_0 + 8y_1 + 5y_2)\right] \\ &= \frac{1}{3} h(y_0 + 4y_1 + y_2) \end{aligned}$$

The 5, 8, minus one rule cannot be used for moments. The first moment of the area between the first and second ordinates ($A_{1\text{-}2}$) about the first ordinate is given by the *3, 10, minus one rule*:

$$M_1 = \frac{1}{24} h^2 (3y_0 + 10y_1 - y_2)$$

These two Simpson's rules are at times convenient, but are less accurate than the first and second rules.

1-4.6.2 Simpson's Rules for Any Number of Ordinates. Simpson's rules can be combined one with another to derive rules for numbers of ordinates for which the first two rules do not apply. For example, the first rule can be used for 3, 5, 7, 9, ... ordinates, and the second rule for 4, 7, 10, ... ordinates. A rule can be deduced for six ordinates as shown below:

$$\begin{aligned} A_{0-3} &= \frac{3}{8} h(y_0 + 3y_1 + 3y_2 + y_3) \\ A_{3-5} &= \frac{1}{3} h(y_3 + 4y_4 + y_5) \\ A &= A_{0-3} + A_{3-5} = h\left(\frac{3}{8}y_0 + \frac{9}{8}y_1 + \frac{9}{8}y_2 + \frac{3}{8}y_3 + \frac{1}{3}y_3 + \frac{4}{3}y_4 + \frac{1}{3}y_5\right) \\ &= \frac{1}{24} h(9y_0 + 27y_1 + 27y_2 + 17y_3 + 32y_4 + 8y_5) \end{aligned}$$

This is not the only rule suitable for six ordinates. By skillful use of the 5, 8, minus one rule, a rule with less awkward multipliers can be deduced:

$$\begin{aligned} A_{0-3} &= \frac{1}{12} h(5y_0 + 8y_1 - y_2) \\ A_{1-4} &= \frac{3}{8} h(y_1 + 3y_2 + 3y_3 + y_4) \\ A_{4-5} &= \frac{1}{12} h(-y_3 + 8y_4 + 5y_5) \end{aligned}$$

$$\begin{aligned} A &= A_{0-1} + A_{1-4} + A_{4-5} \\ &= h\left(\frac{5}{12}y_0 + \frac{25}{24}y_1 + \frac{25}{24}y_2 + \frac{25}{24}y_3 + \frac{25}{24}y_4 + \frac{5}{15}y_5\right) \\ &= \frac{25}{24} h(0.4y_0 + y_1 + y_2 + y_3 + y_4 + 0.4y_5) \end{aligned}$$

Substituting the same values for ordinates y_0 through y_5 in each rule will verify that they are equivalent. Rules deduced in this manner can be used in the general forms described in Paragraph 1-4.4.4.

1-4.7 Other Integration Rules. Simpson's rules and the trapezoidal rule are satisfactory for most manual calculations. The *Newton-Cotes'*, *Tchebycheff's*, and *Gauss'* rules are more accurate, but require more tedious manual calculations. These rules are described in most general naval architecture texts, such as *Basic Ship Theory* by K.J. Rawson and E.C. Tupper, or *Muckle's Naval Architecture* by W. Muckle and D.A. Taylor.

1-4.8 General Notes For Numerical Integration. The numerical integration rules presented have relative advantages and disadvantages. When time and/or access to high-speed computers permits, the salvage engineer may select the optimum integration rule for a well-defined curve. For curves where ordinates are tabulated for only certain stations, a rule appropriate to that number and spacing of stations must be adopted. Some generalizations about the applicability of integration rules are listed below:

- The trapezoidal rule uses constant ordinate spacing and simpler multipliers than the other rules. Any number of ordinates can be used. The rule can accommodate half-stations at any point, and the multipliers for half-stations are easily derived. For a single integration (area calculation) of a gentle curve, the trapezoidal rule is nearly as accurate as the Simpson's rules, but progressively greater errors are introduced on successive integrations (for moments and moments of inertia).

- Simpson's rules and the trapezoidal rule include the common interval as part of the common multiplier and can therefore calculate areas or volumes, moments, centroids, and second moments of area (single, double, and triple integrations) directly.

- Simpson's rules are the most commonly used integration rules because they are more accurate than the trapezoidal rule, but simpler to use than the more accurate Newton-Cotes', Tchebycheff's, and Gauss' rules.

- Simpson's rules exactly integrate first-, second-, and third-order curves. Successive integrations produce progressively higher order curves: the curve of area under a second-order curve is a third order curve, and the curve of the moment of areas is then a fourth-order curve. Simpson's rules will therefore exactly calculate the first moment of a second-order curve, or the second moment of a first-order curve. Calculating the second moment of a second-order or higher curve involves integrating a fourth-order equation, so some error is introduced even for a parabolic curve. Additional error may arise for an arbitrary curve. Experience has shown that Simpson's rule calculates moments and second moments of relatively smooth, continuous curves—such as those describing ship forms—accurately if a sufficiently close station spacing is used.

- An even-ordinate Simpson rule is only marginally more accurate than the next lower odd-ordinate rule; odd-ordinate Simpson rules are therefore preferred, and almost universally used in salvage.

1-4.9 Integration of Discontinuous Curves. The integration rules discussed are applicable to continuous curves. The area under a discontinuous curve can be obtained by applying appropriate rules to the portions of the curve between discontinuities and summing the areas. For curves with large numbers of closely spaced discontinuities, it is simpler to divide the curve into segments at the discontinuities, approximate each segment by a rectangle, triangle, or trapezoid, calculate the area of each segment, and sum the areas to find the total area. The centroid of each segment can be calculated or estimated. Moments, second moments, and the centroid of the entire area can be calculated by summing the products of each area and the lever arm from its centroid to a selected axis in a tabular format. Replacing a segment of the curve between discontinuities (stations) with a horizontal line at a value equal to the average ordinate creates a rectangle with area equal to the area under the curve between the two stations. If the curve between stations can be reasonably approximated by a straight line, a horizontal line intersecting the curve midway between stations has a y value equal to the average ordinate. Repeating this process along the length of the curve creates a *stepped curve*. If the discontinuities, and subsequent stations, are evenly spaced, the curve can be integrated by a modification of the trapezoidal rule:

$$A = \int y\,dx = h\Sigma_1^n y_n$$

$$M_{YY} = \int xy\ dz = h^2\Sigma_1^n (s_n - 1/2)y_n$$

$$I_{YY} = \int x^2 y\ dx = h^3\Sigma_1^n (s_n - 1/2)^2 y_n$$

where:

A = area under a curve between stations 0 and n
M_{YY} = first moment of area about axis YY
I_{YY} = second moment of area about axis YY
h = common interval
s_n = number of stations from axis YY (or integration start point) to station n
y_n = height of the mid-ordinate between stations n and n-1

Weight distribution curves for ships are usually drawn assuming a constant weight distribution between stations as stepped curves. The addition of the continuous buoyancy curve and stepped weight curve creates a discontinuous load curve. The load curve is usually *stepped* as described above to facilitate integration along its length to define the shear curve. Alternatively, the buoyancy curve can be stepped before summing with the weight curve. A stepped 10-segment (11-ordinate) buoyancy curve can be constructed from standard Navy 21-station Bonjean's Curves by taking unit buoyancy calculated from section areas for odd station as the average unit buoyancy for segments bounded by even stations—unit buoyancy for segment 0–2 is based on section area for station 1, that for segment 2–4 on the area for station 3, etc. Example 1-4 includes an integration of this type.

1-4.10 Calculation of Hull Properties. Various integrations of a ship's hull form are used to determine properties such as displacement, locations of centers, tons per inch immersion, etc., known collectively as *functions of form*, *hydrostatic functions*, or *hydrostatic data*. Waterlines, buttocks, and stations of lines drawings are spaced to support numerical integration, usually by Simpson's or the trapezoidal rules. Halfbreadths (offsets) taken along the length of a waterline provide ordinate values to define the waterplane shape; halfbreadths taken at different waterlines at the same station provide ordinate values to define the station shape. Because ships are symmetrical about the centerline, integrations are customarily performed for one side of the section or waterplane only, and **doubled** to give the total area or moment.

When working from offsets, sectional areas are usually calculated by vertical integration on horizontal ordinates from the centerline. An integration up to a waterline gives section area corresponding to that waterline. Integrating the curve of areas along the ship's length gives volume of displacement; the centroid of the volume is the center of buoyancy.

Waterlines are integrated along the ship's length to determine area of the waterplane, location of the centroid of the waterplane (center of flotation), and moment of inertia of the waterplane about the centerline and about a transverse axis through the center of flotation. From these properties, tons per inch immersion, location of the metacenter, etc., can be calculated. Displacement volume can be calculated by taking waterplane areas as ordinates and integrating vertically.

Longitudinal position of the center of buoyancy (*LCB*) is obtained by longitudinal integration of the sectional areas. Height of the center of buoyancy (*KB*) can be obtained by vertical integration of waterplane areas, or by calculating a vertical moment of area for each section. The sum of all the vertical area moments divided by the sum of the sectional areas gives *KB*. Integrations of this form are included in Example 1-4 and Appendix F.

1-4.10.1 Functions of Form. Functions of hull form are usually calculated for each waterline so they can be plotted as a function of draft as the ship's *Curves of Form*, also called *Hydrostatic Curves*, or *Displacement and Other Curves* (*D & O Curves*). Figure FO-2 is a reproduction of the curves of form for an FFG-7 Class ship. Hydrostatic data is also recorded in the Functions of Form Diagram (Figure B-1) for Navy ships and Hydrostatic Tables (Figure B-2) for commercial vessels. The salvage engineer may be required to calculate hydrostatic data when curves of form or other documents are not available or for a casualty in an unusual condition. Whether functions of form are calculated for a complete range of drafts or for only a few selected drafts depends on the form of the ship and the nature of information required by salvors. Manual calculations are best performed on organized tabular forms called displacement sheets.

Table 1-3. Appendage Allowances.

Ship Type	Appendage allowance: Δ_{APP}/Δ_{FL}
Single-screw, small combatant with keel sonar dome[1]	0.0167
Twin-screw, small combatant with keel sonar dome[1]	0.0200
Single-screw, small combatant with bow sonar dome[1]	0.0049
Twin-screw, small combatant with bow sonar dome[1]	0.0060
Twin-screw amphibious warfare ships with well decks[1]	0.0106
shell plating only	0.0057
all other appendages	0.0049
Twin-screw LST[1]	
without bow thruster	0.0024
with tunnel bow thruster (negative appendage)	0.0014
Single-screw merchant ships and auxiliaries of ordinary form, less than 5,000 tons full load displacement	0.0075
shell plating only	0.0060
all other appendages	0.0015
Single-screw merchant ships and auxiliaries of ordinary form, 5,000 to 15,000 tons full load displacement	0.0050
shell plating only	0.0040
all other appendages	0.0010
Single-screw merchant ships and auxiliaries of ordinary form, greater than 15,000 tons full load displacement	0.0025
Twin-screw merchant ships and auxiliaries of ordinary form	0.0081
shell plating only	0.0035
all other appendages	0.0046
VLCC, ULCC, very large bulk carriers	0.0015

Source: [1]Jamestown Marine Services, 1990, unpublished; based on data from 22 hull types entered into ship data files for the NAVSEA *POSSE* Program

1-4.10.2 Appendage Displacement. Volumes and displacements (buoyancies) based on section areas taken from Bonjean's Curves do not include appendage volume/ displacement, although sectional areas from some Bonjean's Curves include shell plating. If known, appendage displacements can be added to the integrated displacement; effect on *LCB* can be determined by moment balance. When appendage buoyancy is unknown, appendage displacement can be estimated as a fraction of full load displacement, called an *appendage allowance*. Appendage allowances vary with ship size, type, and configuration. Warships generally have more and larger appendages than auxiliaries or commercial vessels. Vessels with high power-to-size ratios have larger screws and rudders than lower powered vessels; appendage allowance increases with the number of screws. Large bow sonar domes on combatants are faired into the hull, and are included in Bonjean's Curves and offsets; keel-mounted domes are appendages. For a given ship type and configuration, appendage allowance generally increases as size decreases. Approximate appendage allowances for different ship types are given in Table 1-3.

Appendage displacement is essentially constant with draft, as most appendages (except shell plating) are low on the hull and will be emerged only by extremely low drafts. Once determined, appendage displacement can be added to the integrated displacement for any draft that covers the appendages to determine total displacement. Shell plating displacement can be adjusted for drafts less than full load by assuming that one-half of the shell plating volume is concentrated in the bottom third of the draft range, and the remaining volume is evenly distributed over the upper two-thirds of the draft range. It is usually safe to assume that *LCB* for the displacement with appendages is virtually the same as that for the integrated (without appendages) displacement.

1-4.10.3 Station Spacing. In full-bodied ships (low-speed general cargo, large tankers, bulk carriers, etc.) the lengths of the waterlines between stations in the midbody are nearly straight lines. In many modern full-bodied ships, the waterlines over the midbody are, in fact, straight lines, forming a parallel midbody. Integration on 10 equal divisions of length (11 stations, 0-10) is sufficiently accurate for most purposes. If the curvature of the waterlines increases sharply near the ends of the ship, half-spaced stations can be inserted to increase accuracy, for example, at stations ½, 1½, 8½ and 9½.

Accuracy can be increased by reducing the station spacing throughout the length of the curve. This increases the number of calculations to be performed, but avoids determining additional multipliers and may be simpler to program for computer calculation. For ship calculations, offsets are usually tabulated for either 11 or 21 basic stations (10 or 20 equal divisions), with half-stations as necessary. Offsets for Navy ships are normally tabulated for 21 basic stations, although additional tables may be prepared for very close station spacing. Offset tables for 2-foot station spacing are available for the FFG-7, for example. Even when 21-station offset tables or Bonjean's Curves are available, integration on 11 stations is sufficiently accurate for most hull volume calculations on any smooth hull form, including fine-lined warships.

1-4.10.4 Full Sections. In full, relatively flat-bottomed sections, special care must be taken in calculating the area from the base to the lowest waterline to avoid error. Figure 1-11 shows a section near midships where the turn of the bilge fairs into a straight line (the rise of floor line) at point *A*. If the entire area below *CD* is calculated using horizontal ordinates from the centerline, very close ordinate spacing must be used to avoid error because of the rapid change of form in the shell line. The area below *CD* can be calculated accurately using vertical ordinates from *CD*, with half-spaced ordinates inserted near the outboard end, or by dividing the area into two segments, as shown. The area *KABC* is a trapezoid whose area can be calculated accurately when the position of *A* and rise of floor can be determined. The area *ADB* can be obtained by using Simpson's rule, either with horizontal ordinates measured from *AB*, or with vertical ordinates measured from *BD*.

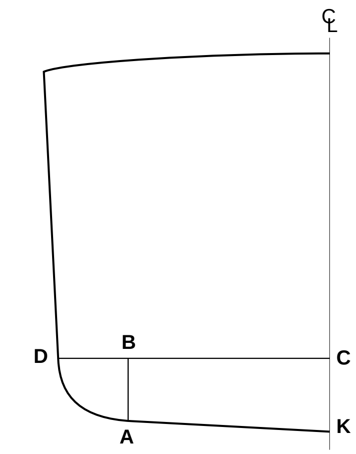


Figure 1-11. Calculating Sectional Area Below the Lowest Waterline.

1-4.10.5 Lowest Waterlines. When displacement volume is calculated by vertical integration of waterplane areas, the volume under the lowest one or two waterlines is calculated separately. Since the form of the ship changes so rapidly near the keel, the volume under the lowest one or two waterlines is calculated by integrating sectional areas along the ship's length. This volume is added to the volume determined by integrating waterplane areas from the lower waterlines upward to obtain the total volume of displacement.

1-4.10.6 Ends of Full Hull Forms. On very full hulls, such as *spoon-bowed* barges, large tankers (VLCC, ULCC), and bulk carriers, the parallel midbody extends nearly to the ends of the ship, where it joins to a short forebody or afterbody with steep or sharply curving lines. The aft ends of the lower waterlines of many fine-lined ships also curve sharply. If the ordinate adjacent to the end ordinate is some distance away from the end of the parallel midbody, the curve from this ordinate to the end ordinate (which is 0 or very small) assumed by Simpson's rules or the trapezoidal rule will fall well inside the actual waterline as shown in Figure 1-12. This will cause a serious underestimation of area for the end sections that will lead to even greater errors in calculations of moments and second moments about axes near midships because of the long lever arms. Intermediate stations should be inserted so that there are ordinates near the ends of the parallel midbody and at least one or two ordinates in the forebody and afterbody. Alternatively, waterplane areas for the midbody, forebody, and afterbody can be calculated separately and summed. The midbody area can be treated as a rectangle or integrated by a 3-ordinate Simpson or trapezoidal rule; the midbody and forebody areas can be calculated by any convenient rule with appropriate ordinates.

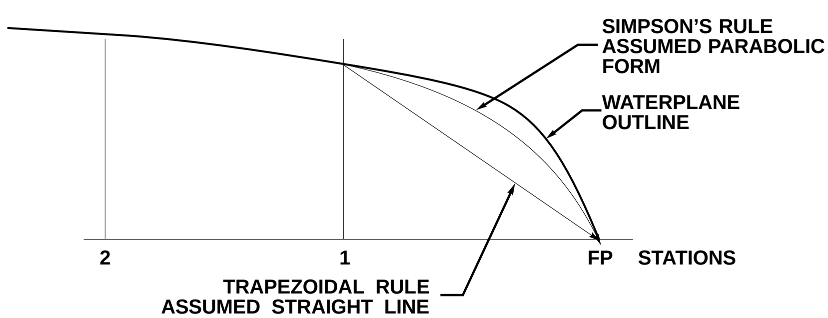


Figure 1-12. Inherent Integration Error in Full Waterlines.

1-4.10.7 Tank and Compartment Volumes. A compartment's molded volume is greater than its floodable volume (the volume of liquid that can be contained), because of the volume occupied by fittings and structure. Floodable volumes of filled holds, machinery spaces, living spaces, etc., are estimated from molded volumes by use of permeability factors, as explained in Paragraph 1-9.1.1. Framing, sounding tubes, sea chests and similar structures in ordinary *skin* tanks typically occupy about 2¼ to 2½ percent of the molded volume in double-bottom tanks, about 1 percent in cargo tanks (i.e., permeability of **empty** tanks is 97½ to 97¾ percent, and 99 percent, respectively). Heating coils, if fitted, usually occupy an additional ¼ percent of the molded volume. Flush tanks lie entirely within the ship's framing and are externally stiffened, so floodable volume, or capacity, is essentially equal to molded volume. To calculate volumes and centroids of flush tanks, offsets are taken to the inner surface of the tank, rather than the hull molded surface. Bale capacity of holds is calculated from offsets taken from sections showing the line of cargo battens, line of the bottoms of deck beams, and the top of the hold ceiling (above the inner bottom) including any gratings, with deductions for stanchions and other obstructions. Grain capacity is the molded volume, less the volume of structure, hold ceiling, and shifting boards.