ME 4811

Lab #7: The Linear Quadratic Servomechanism

We want to consider the problem of designing a stabilizer for the lateral mode of an aircraft. This provides an example of a multivariable control system with two forcing functions (the aileron and rudder signals) and three outputs (yaw rate, roll angle, and side slip velocity). In order to carry out a coordinated turn; i.e., without sideslip, it is usually necessary to deflect both the aileron and the rudder. Furthermore, the system is interacting in that both sets of control surfaces give rise to rolling and yawing motions.

The state variables for the problem are defined as follows:

x_1 = dimensionless sideslip velocity,
x_2 = roll or bank angle,
x_3 = roll rate,
x_4 = yaw rate,
x_5 = aileron angle,
x_6 = rudder angle.

The two forcing functions are the electrical inputs u_1 and u_2 to the electrohydraulic control surface actuators, which are assumed to have transfer functions with a first order lag (time constant) of 0.1 seconds.

The state equations are:

$$\dot{x}_1 = -0.09x_1 + 0.097x_2 - x_4,$$
$$\dot{x}_2 = x_3,$$
$$\dot{x}_3 = -5.43x_1 - 0.686x_3 + 3.62x_4 + 2.87x_5 + 0.638x_6,$$
$$\dot{x}_4 = 0.56x_1 - 0.122x_4 + 0.127x_5 + 0.459x_6,$$
$$\dot{x}_5 = -10x_5 + 10u_1,$$
$$\dot{x}_6 = -10x_6 + 10u_2.$$

Unlike the classical LQR design, extra state variables must now be added to correspond to the reference model or tracking input. For example, if the design is to be for a step input then the seventh equation is

$$\dot{x}_7 = 0.$$

If the design is for a ramp input, then two extra variables are required:

$$\dot{x}_7 = x_8 \text{ and } \dot{x}_8 = 0,$$

where x_7 is the reference variable. Now say that the desired roll response here corresponds to a quadratic transfer function with a damping ratio of 0.7 and an undamped natural frequency of 3 radians per second. In other words, the actual roll rate response of the aircraft should approximate that of x_7, where

$$\ddot{x}_7 + 4.2\dot{x}_7 + 9x_7 = 0,$$

or

$$\dot{x}_7 = x_8,$$
$$\dot{x}_8 = -9x_7 - 4.2x_8,$$

and these equations are used to augment the previous set of state equations.

The performance criterion is taken as

$$\int_0^\infty \left[(x_7 - x_3)^2 + a x_1^2 + a_1 u_1^2 + a_2 u_2^2 \right] dt,$$

which places emphasis on three things,

- the deviation of the actual roll rate from the desired,
- minimization of the sideslip velocity, and
- restricting the actuator signals.

The relative importance of the four terms in the performance index is adjusted by choice of the parameters a, a_1, and a_2.

Do the following:

1. Set-up the optimization problem and give expressions for all matrices involved.
2. Choose a=1, a_1=0.5, a_2=0.1 and use Matlab to obtain the actuator signals u_1 and u_2.
3. Typically, all state variables are available for measurement with the exception of the sideslip velocity x_1. Design an observer to estimate x_1.
4. Repeat the design of step (2) with different choices for a, a_1, a_2. What are the resulting closed-loop poles? How do the gains in the control laws change as you change a, a_1, and a_2?