ME 4811

Lab #2: Control Law Design

Consider the Phoenix equations of motion of Lab #1. Assume that the bow rudder is slaved to the stern rudder so that

$$\delta_b = -\delta_s \ ,$$

in other words if δ is the deflection of the stern rudder, the bow rudder assumes a deflection of $-\delta$. In this way we get a single input system. Write this in state space form,

$$\dot{x} = Ax + Bu \ ,$$

where x is the state variables vector $[v, r, \psi, y]$ and u is the control vector δ.

Do the following:

1. Give the transfer function between lateral position y and rudder angle δ. Verify that the eigenvalues of the A matrix are in both cases the same as the poles of the transfer function.

2. Find the controllability and observability matrix. Compute the matrix product `OC.` This product is the same for every definition of state variables for a given system.

3. Diagonalize the A matrix of the original system by finding its eigenvalues and eigenvectors. Verify that the product $T^{-1}AT$ where T is the matrix of eigenvectors of A is a diagonal matrix with the eigenvalues of A in its diagonal. Write the normal coordinate form of the state equations.

4. Investigate the controllability of the system in response to rudder deflections. Compute the rank of the controllability matrix of the original system. Is it full?

5. Assuming that the system is controllable, do the control design of the original system by pole placement. Set the poles so that they all have approximately the same time constant. How do the gains change for different control time constants? Make a graph and select a time constant and gains combination that make sense to you physically.

6. Evaluate your control design with numerical simulations. Assume that the ship is initially located off the desired track by one ship length port side with zero heading. Keep the sine and cosine terms in the simulations and plot all variables of interest, at minimum the rudder angle and a geographical x-y plot. Does y converge to zero? Limit the rudder angle to ± 0.4 radians. If needed, change your gains and/or poles selection so that the response looks nice. Test another response where the initial y is zero but the ship is initially pointing 30 degrees starboard side.