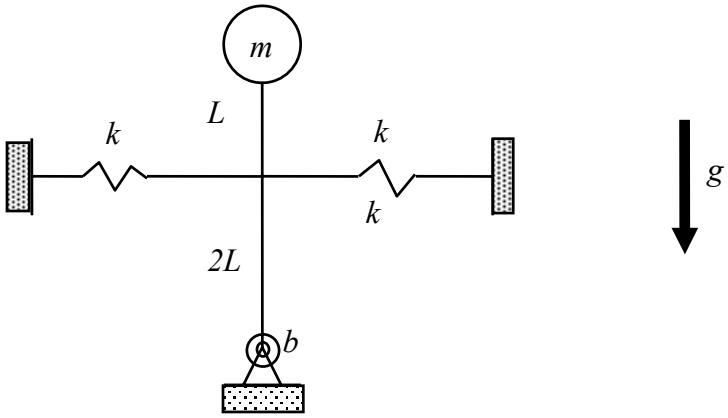
ME 4811

Lab #10: Nonlinear Systems

Consider the inverted pendulum supported by the two springs with constant k and the rotational damper with constant b, as shown in the figure. Assume that as the pendulum deflects by an angle θ both springs remain horizontal. You may also assume that the bar has negligible mass. Use the numerical values m=1, L=1, g=10, and b=3.



Do the following:

1. Show that the equation of motion, keeping all nonlinear terms, for the pendulum is
$$9mL^2\ddot{\theta} + b\dot{\theta} + (8kL\cos\theta - 3mg)L\sin\theta = 0.$$

2. By setting all time derivatives equal to zero calculate all possible values for the equilibrium angle θ as a function of k>0. Plot your results for θ versus k.

3. Linearize the system and assess the stability of your equilibrium solutions.

4. Confirm your results using numerical simulations. Pick two representative values for k, one "small" and one "large". Use appropriate initial conditions for your integrations to show convergence to the appropriate solution.

5. Comment on the usefulness and limitations of linear theories. Do nonlinear and linearized system results contradict or complement each other?