

## ME 4811

### Lab #1: Simulations

The equations of motion used to simulate the dynamic behavior of the NPS Autonomous Underwater Vehicle “Phoenix” in the horizontal plane are as follows:

Sway equation 
$$(m - Y_v \dot{v}) \dot{v} - (Y_r - mx_G) \dot{r} = Y_v v + (Y_r - m)r + Y_{\delta_s} \delta_s + Y_{\delta_b} \delta_b$$

Yaw equation 
$$(mx_G - N_v \dot{v}) \dot{v} - (N_r - I_z) \dot{r} = N_v v + (N_r - mx_G)r + N_{\delta_s} \delta_s + N_{\delta_b} \delta_b$$

Turning rate 
$$\dot{\psi} = r$$

Inertial position rate 
$$\dot{y} = \sin \psi + v \cdot \cos \psi$$

All variables in these equations are assumed to be in nondimensional form with respect to the vehicle length (7.3 ft) and constant forward speed (approx. 3 ft/sec). The vehicle weighs 435 lbs and is neutrally buoyant. This is not needed in the calculations that follow, but it gives you an idea of the physical system. Time is become nondimensional so that 1 second represents the time that it takes to travel one vehicle length. In the equations of motion, the variables are defined as:

$v$	lateral (sway) velocity
$r$	turning rate (yaw)
$\psi$	heading angle
$y$	lateral deviation (cross track error)
$\delta_s$	stern rudder deflection
$\delta_b$	bow rudder deflection

The rest are constants,  $m$  is the mass,  $I_z$  is the mass moment of inertia with respect to a vertical axis that passes through the vehicle’s geometric center (amidships),  $x_G$  is the position of the vehicle’s center of gravity (positive forward of amidships), and the remaining terms are the so-called hydrodynamic coefficients. Nondimensional values for the coefficients are given in the following table:

$m = 0.0358$	$Y_{\delta_b} = 0.01241$
$I_z = 0.0022$	$N_{r \dot{v}} = -0.00047$
$x_G = 0.0014$	$N_{v \dot{v}} = -0.00178$
$Y_{r \dot{v}} = -0.00178$	$N_r = -0.00390$
$Y_{v \dot{v}} = -0.03430$	$N_v = -0.00769$
$Y_r = 0.01187$	$N_{\delta_s} = -0.0047$
$Y_v = -0.10700$	$N_{\delta_b} = 0.0035$
$Y_{\delta_s} = 0.01241$	

Figures 1 and 2 show a picture of the vehicle and the definition of the coordinate systems used in the equations of motion.

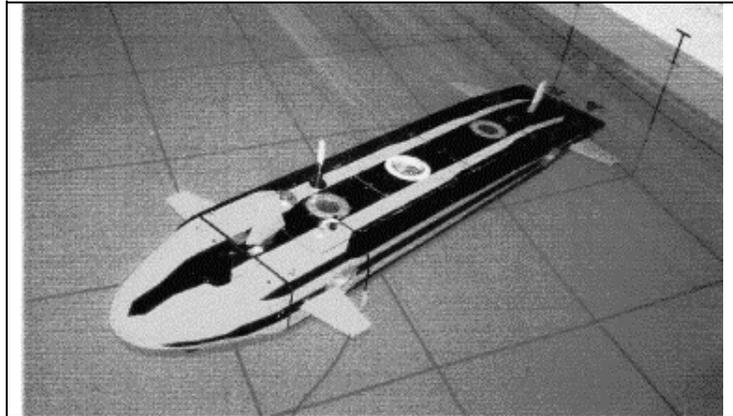


Figure 1:  
Picture of the “Phoenix” vehicle

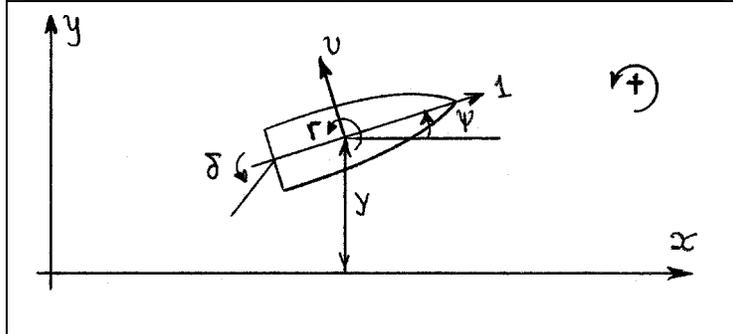


Figure 2:  
Geometry and axes definitions

**Do the following:**

1. Select as state vector  $x = [v, r, \psi, y]$ , control vector  $u = [\delta_s, \delta_b]$ , and write the state space equations, assuming small angles  $\psi$ ,

$$\dot{x} = Ax + Bu .$$

Give the values for the  $A$  and  $B$  matrices.

2. Using matrix algebra, compute the transfer functions between lateral position  $y$  and either stern rudder deflection  $\delta_s$  or bow rudder  $\delta_b$ . What are the open loop poles and zeros in each case? What is the physical significance of open loop poles that are equal to zero (if any)?
3. Draw the block diagram of the system, keeping all sine and cosine terms, and simulate:
  - 15 degrees of positive bow rudder, stern rudder at zero.
  - 15 degrees of negative stern rudder, bow rudder at zero.
  - 15 degrees of positive bow rudder, 15 degrees of negative stern rudder.
 See item #5 for the graphs to submit.

4. Simulate the system using Euler's integration and Matlab. Use the same conditions as #3. Do your results agree with #3? For the remaining of these assignments, use either the Simulink block diagram or the Matlab code as the basis for your simulations. Keep in mind that, based on previous experience, Matlab code tends to be easier to debug! Submit your graphs as in item #5.
5. For both questions 3 and 4, plot a geographical  $(x,y)$  plot for the ship's position. For this you'll need the additional equation for the rate of change of  $x$  which is,

$$\dot{x} = v \cos \psi - r \sin \psi .$$

Allow enough time to complete a full turning circle. Comment on the effectiveness of the various rudder deflections on the turning diameter.