

## CONTROLLER / OBSERVER CANONICAL FORMS

Consider the transfer function

$$\frac{y}{u} = \frac{s^2 + s + 2}{s^3 + 3s^2 + s + 1}$$

Find its state-space representation:

```
>> num=[1,1,2];den=[1,3,1,1];
>> [a,b,c,d]=tf2ss(num,den);
>> a
a =
-3      -1      -1
 1       0       0
 0       1       0

>> b
b =
 1
 0
 0

>> c
c =
 1       1       2
```

Check the eigenvalues of the  $A$  matrix:

```
>> eig(a)
ans =
-0.1154 + 0.5897i
-0.1154 - 0.5897i
-2.7693
```

Confirm they match the poles of the transfer function:

```
>> roots(den)
ans =
-2.7693
-0.1154 + 0.5897i
-0.1154 - 0.5897i
```

OBSERVER canonical form is:

$$\begin{bmatrix} \dot{x}_1 \\ x_2 \\ \vdots \\ x_3 \end{bmatrix} = \begin{bmatrix} -3 & 1 & 0 \\ -1 & 0 & 1 \\ -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} u \quad \text{and} \quad y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

CONTROL canonical form is:

$$\begin{bmatrix} \dot{x}_1 \\ x_2 \\ \vdots \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -1 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u \quad \text{and} \quad y = \begin{bmatrix} 2 & 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

#### Properties:

1. Both forms have the  $A$  matrix in a companion form.
2. OCF has full  $B$ -matrix and simple  $C$ -matrix.
3. CCF has simple  $B$ -matrix and full  $C$ -matrix.

Both forms produce the same transfer function:

```
>> accf=[0,1,0;0,0,1;-1,-1,-3];bccf=[0;0;1];cccf=[2,1,1];
>> aocf=[-3,1,0;-1,0,1;-1,0,0];bocf=[1;1;2];cocf=[1,0,0];
>> [numccf,denccf]=ss2tf(accf,bccf,cccf,0)
```

numccf =

```
0      1.0000      1.0000      2.0000
```

denccf =

```
1.0000      3.0000      1.0000      1.0000
```

```
>> [numocf,denocf]=ss2tf(aocf,bocf,cocf,0)
```

numocf =

```
0      1.0000      1.0000      2.0000
```

denocf =

```
1.0000      3.0000      1.0000      1.0000
```